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HYPERACOUSTICS

DIV. I.

HYPERACOUSTICS

BY JOHN L. DUNK

DIVISION I

SIMULTANEOUS TONALITY

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PREFACE

The fascination which the art of music exercises upon many theoretically minded persons is not easy to define in strict terms of practical profit.

Although the primary attraction may possibly arise from ideas of aiding the art, it is much more likely to be due to the intuitive recognition of an avenue to the working of the human mind.

It is generally recognised that some acquaintance with musical theory (academic or otherwise) is useful, if not a necessity, to all who practise and understand the art.

But the form that this region of knowledge shall take is not a matter upon which authorities agree.

The problems to be considered range from the purely empirical to the strictly external, practically from æsthetics to mechanics.

It is therefore not unusual for the student to turn to the science of "Sound" (known generally as "Acoustics"), which is concerned largely with problems regarding musical tone from various points of view.

The physical division of this science is that part of the general theory of vibro-mechanics which is limited to values of auditory interest: this somewhat illogical selection is justified by the preponderating interest attaching to audible sound.

It has been said that the proper study of mankind is man.

Certainly, modern science is greatly concerned with all that pertains to the genus "Homo Sapiens."

Investigations turn upon two broad divisions of inquiry: What man is, and what man does.

The varied activities of the human race include the practice of the arts, ranging from the useful to the so-called "fine" arts, the latter category including music.

The widespread practice and comprehension of the art (in composition, performance, and audition) justifies the amount of interest taken in these particular doings of man by observers of a scientific turn of mind.

Apart from the artistic side, which claims so many votaries, the peculiar nature of music (its intangible media, its formulative and numerical basis, etc.) presents some profoundly interesting problems to all those who incline towards a scientific outlook upon the world around them.

A broad examination of musical works in general reveals an appearance of law and system which may or may not be justified on closer scrutiny; it is, however, sufficiently striking to attract attention and to invite investigation.

Again, from the opposite point of view, the elementary relation of acoustical principles to much that is essential in musical practice is fairly obvious.

Hence there is reasonable ground for the assumption (as a basis of study in the absence of other conditions) of an ideal region of knowledge which shall link the two extremes together.

The unknown matters comprised in this region may by assumption be regarded as ground for a science to which the name "Hyperacoustics" may be applied.

The extent to which these preliminary ideas may afterwards be found to hold, to require modification, or to be abandoned, is a question which each individual student may attempt to answer at various stages in the investigation.

The justification for undertaking such an investigation is obviously not to be stated in a preface, but if it be admitted that knowledge consists in knowing why we do not know certain things as well as what we do know, the gain to precise thinking is worth the effort.

The art of music is an important factor in the lives of many people. As an attribute of humanity it is embraced in the comprehensive realm of Natural Philosophy, and becomes therefore a proper subject for scientific scrutiny, not as an art, but purely in the aspect of being something that profoundly concerns and interests a considerable section of humanity.

The facilities of the present day offer every inducement to those who would study music as an art.

At no previous epoch in the world's history have there been so many avenues of approach open to the average individual.

Teachers and schools abound, together with opportunities for the performance and audition of the best works.

There are, in addition, avalanches of periodicals and shoals of

literature dealing with almost every conceivable phase of the subject, together with the most ingenious devices of instrumental technicality, as well as automatic aids to performance.

To these varied and compliable means of entrance into the practical realm of music, every authority commends the would-be aspirant desiring to know and experience the Elysium of music as an art; and, by the exercise of sheer ardour and diligence, who may not enter therein?

There is, indeed, no royal road to learning, but under sympathetic and mutual help, obstacles that appear insurmountable have a way of decreasing.

But the approach to a purely scientific discussion of musical experiences, and their influences upon the thoughts and habits of mankind, opens up quite another standpoint, in which the non-musical may join equally with the musical.

The first step is the recognition, from a purely impersonal standpoint, of such an aspect, prompting some such question as to the reason why a large section of humanity is, and continues to be, so interested in certain successions of sounds and their concomitants, often with so little material justification.

The right of a scientific observer to tread the sacred ground of an art may be challenged, and such a one is certain to be called upon to justify his action, upon the basis of any expectation of good to himself or humanity, which may be considered likely to result from such researches.

This is a question which must be left unanswered at present.

Assuming that it is possible for the phenomena included under the title of " music " to come under scientific consideration, and presuming this view to be included wholly or in part in the aims of the multitude of investigators, it becomes most necessary to attempt definition of the realm it is intended to consider.

Music is a manifestation of a conscious and sentient humanity, and a resultant of activity. As such, it is largely an introspective experience, and eludes examination.

But in carrying out the manifestation, certain material conditions are involved, and, since some or all of these are also concerned in other branches of investigation, research tends towards a definite avenue of approach.

The science of Acoustics (employing the term in the original meaning of Joseph Sauveur, and not restricting it, as now usual

in Britain, to the science of sound in buildings) is a division of the general inquiry which specifically investigates sound.

It falls naturally into three great divisions: Physical, Physiological, and Psychological; thus covering a range extending from the purely mechanical to the purely sentient.

Particular interest is directed towards the class of phenomena concerned in musical manifestation, which in consequence of their superlative interest and numerical tractability, necessarily form a large portion of the subject matters under consideration.

Between the region of phenomena (undefined) comprised in the science of Acoustics, and the experiences of music considered as phenomena, there appears a great gulf, which invites attempts to bridge.

Many investigators have essayed to connect the two divisions, both in the quest of pure knowledge, and also with the aim of aiding the art and practice.

In order to form a concept of this vast and mysterious region, a name is required. This (to the writer's knowledge) has not yet been conferred, owing to the loose definity of purpose and lack of cohesion in the investigation, especially in the preliminary stages.

Each investigator attacks the region and states the problems in terms that seem best to himself.

This may indeed be an advantage in providing several independent avenues of approach, but it does lead to a want of understanding, and even sympathy, among the assailants; and certainly the results so far attained appear to lack that cohesion which makes scientific progress possible, and which practically facilitates the comprehension by others of what has really been effected.

There is no reason to expect that an investigation will answer the actual questions propounded; a mere glance at history shows that. (How much nearer are we to a knowledge of what gravity is than Newton? and yet what vast developments have taken place in cosmic mechanics upon the lines he laid down.)

Before giving way to any feeling of disappointment, the investigator would do well, therefore, to ask if his question was one that could be legitimately and pertinently asked. This particularly applies to the fact that the criterion of what is and what is not music, like the nature of gravity, escapes scientific scrutiny.

There is no doubt that any name which may be proposed to embrace the aforementioned vast indefinity must unavoidably possess many disadvantages, whereby it is laid open to stringent criticism.

The "gulf" has two sides, and can be approached either by working forwards from the material aspect of Acoustics, or backward from the experiential aspects of music.

However, in the present investigation it is the purely scientific side that is emphatically insisted upon.

Hence the name "HYPERACOUSTICS" may be proposed, as indicative not only of something beyond, but also of a presumption requiring justification as to the existence and rationality of something beyond the known facts of acoustics.

Having proposed a name for the region of presumed scientific interest, it is necessary to decide what shall be included and what excluded from the pale.

Too extensive a category would tend to dilute the subject and weaken the efforts of investigators therein, while the evil effects of arbitrarily trammelling the scope of a scientific inquiry by limited preliminary concepts are only too patent in the history of science.

A somewhat arbitrary division, based upon the general abstracts of experience, may therefore be formulated.

When a particular characteristic is found to inhere generally in a large set of independent experiences, it may be abstracted and dealt with as a thing in itself.

The ideal may never be attained (any more than a pure musical sound free from noise), but it is a definite concept which can be examined as an ideal phenomenon, and the results of such inquiry can be applied to cases in which the abstract characteristic is prominent, without losing touch of the actual circumstances.

This is the general practice in scientific inquiry and exposition, and is to be found stated in various terms in the prefaces of most great works (*e.g.* Thomson and Tait's *Natural Philosophy*, Rayleigh's *Sound*, etc.).

Granting the preceding assumptions, the subject tends to fall into separate divisions or aspects, not actually separable, but conveniently considered so. These are:—

TONALITY: Simultaneous and Successive — the science of musical sound in pitch and quality;

RHYTHM: the aspect of protension in time and "æsthesis";
ORGANISATION: the conditions of constitution in actual musical works;
and lastly: SIGNIFICANCE. Attempted definition of this last must be suspended for the present, but it implies the distinction between music and mere "correct" presentation.

Of these, the present essay is restricted to the aspect of Tonality.

In offering an attempt at an introduction to the subject, one necessary fact, consequent upon the indefinity before referred to, must be emphasised.

When writing upon any scientific subject whose regionality is recognised, there is some basis upon which to work. In compiling a graded course of studies, one is entitled to presume a certain amount of knowledge in the reader at any definite stage in the exposition.

Again, in writing for the technical reader, the alphabet of the particular matter dealt with can be presumed to be known.

What is known as "Popular" writing for the general reader is less easy, and may fatally descend to mere sparkling clap-trap and catch phrases which do more harm than good.

In general writing, therefore, the necessary aim is to strike the happy mean between overwhelming prolixity and abstruse conciseness. This, though admittedly difficult, has to be attempted to the best of a writer's judgment.

At this stage it may be questioned why some common basis or school of thought has not been developed.

Much, of course, has been done, but the results are more applicable to detail matters.

The whole crux of the problem turns upon what constitutes a community of concepts, viz. a language.

The musician has his notations; the mathematician his symbols; the chemist his formulæ; the biologist his genetic and specific system; etc.; all of which have to be learned by any one wishing to enter these regions of interest.

The art of music, considered from what is usually known as its theoretical side, is rich in its terminology; but the scientific aspect of "Hyperacoustics" as yet lacks the general language to connect its concepts with common thought.

One of the greatest workers in the field, Moritz Hauptmann, published his views in 1853 (*The Nature of Harmony and Metre*) —later developed by Otto Baehr in 1882.

Once the contents of his work are assimilated, it will be seen that there is much food for thought therein. But Hauptmann had no language to express directly what he thought, and chose the vehicle of a special philosophy which happens to be by no means easy to grasp. Again, soon after the publication of his work, the purely acoustical aspect received an immense stimulus from the great work of Helmholtz, which tended to overshadow the purely conceptional ideal aimed at by Hauptmann.

It is a somewhat bold and solemn undertaking to attempt to supply this deficiency of language, whose necessity may not even command assent.

The writer who introduces a single new term or symbol at once incurs a debt to his readers for a certain amount of mental energy, and an attempt must be made to honourably liquidate this obligation.

It was hoped that the present work might include a section dealing with the objective foundation in the threefold aspect of Acoustics: Physical, Physiological, and Psychological; and that a historical and bibliographical survey might also be appended, but it is not desirable to overweight these pages; consequently much has had (for the present) to be deleted.

The particular division of inquiry dealt with in these pages is suggested by the following considerations:—

An investigation of musical problems is bound to lead to the obvious puzzle presented by musical experience when compared with the acoustic knowledge attained at any time.

No sooner does the student fasten upon some definitive principle that can apparently be honoured as a "law" than some practical achievement apparently supervenes the limitations laid down thereby.

The dicta of the text-book (although as useful as ever to the tyro) are hardly out of the press before some acknowledged musical empiricist apparently scatters them to the wind.

In the face of these experiences, the student suffers some discouragement, and keenly feels the lack of a basis upon which to formulate a constructive theory.

The pragmatist has no such qualms, since he boldly announces

that the theorist follows the empiricist, and only claims the text-book to be a condensation of practice.

This, however serviceable in practice, fails to satisfy the scientific appetite, and least of all, the student, who is asked to discipline his youthful ardour, talent, and perhaps sheer genius, to something that cannot be maintained as true.

The same problem, under a different aspect, presents itself to the observer of the historical development of music. On the one hand is to be observed early evidences that man knew and appreciated some of the primary harmonic relations leading directly to the Major Common Chord; while on the other, the tendency of practice appeared to neglect the rich and round "major" effect in preference for a model "quasi-minor" style of exposition in scale and melody.

In order to justly appreciate the problem one has to recognise the important principle that acoustic conditions can only determine the course of practice *after* definite attempts have been made.

The actual composition of music, primarily melody, is the attempt of a composer to achieve an aim, viz. an expression, which he clothes to the best of his ability in a particular form.

The test of vitality is the survival of actual works, or the production of forms stimulated by them; and the conditioning of these is due to many circumstances, several extraneous, some being due to national or sentimental idiosyncrasy, but in the main, the moulding influence must be attributed to the persistent operation of the objective acoustic conditions, which outlive the swaying of human passion and fashion.

It is obvious that the acoustic tendencies can have no effect upon music not yet composed. Whenever a composer has consciously endeavoured to be "scientific" he has generally spoiled the composition.

One cannot know or form an estimate of the proportional effect that acoustical conditions can have upon music yet to come. The mechanical problem of "three bodies" is simple in comparison. Consequently, any attempts to peer into the future are generally without real result.

Man must give forth the effort of his artistic activity, must venture forward sustained by his inward power controlled by inhibition, and then, and not till then, can the acoustical and hyperacoustical conditions come into operation.

Synergy, after all, implies pre-formed association upon which formulæ can be plotted under the guidance of æsthetic and certain extraneous conditions.

These persist in consequence of their memorial and emotional vitality; much being possibly due to the association of external conditions occurring at a particular time. All the while that the process was, and is, going on, the ineliminable conditions of tonality were potentially present.

They could not act upon what was uncreate, but the musical actuality having been formed, the harmonic factors slowly tended to conform (and even to trammel) it to the general ter-triadal structure which now forms the fundamental basis of tonality.

We are consequently reaping the result of generations of æsthetic perceptions and active essays. Upon past effort, present law and order (such as it is) rests.

To attempt any prediction of the future development of tonal practice is beyond our scope, although some hypotheses may be examined in the forthcoming pages.

We may perhaps be able to determine some of the persistent acoustic tendencies, but we are to a great extent unable to sort out the potent from the sterile elements inhering in the infinitude of possibilities.

The work of actual musical construction must go on if these factors are to have media to conform to their immutable principles.

Thus it is that animate men and inanimate physics go hand in hand, neither presuming to dictate to the other.

It is a truism of mechanics that a force can exhibit no effect except upon matter. Similarly, synergy determining the aggregation of special groups of associations, out of many so possible, by reason of arbitrary or inevitable conditions, cannot be manifested before such associations are independently brought about.

The series members of a tone are not individually evident to the auditory sense until attention has been drawn to them; therefore their pitch relations, the concordant and discordant intervals, can have no determining effect upon a plan of action not yet carried out.

But after chords have been formed by empirical methods guided by æsthetic perception and judgment, the characteristic chromality inhering in the triadal and regular scalar forms will gradually assert itself until a real sense of chromal perceptivity is

synergetically built up upon the associated groups, leading along definite lines of practice (whose abuses have nothing to do with the general principle).

The formulation of a harmonic system by "trial and error" methods, largely as a by-product of polyphony, is evident from the course that musical development has followed; and it may be frankly conceded that this naïve process appears to be the main source of modern chordance.

Theory of a more or less crude type always accompanies earnest effort, and attempts at all times have been made to foist some scheme of imperfectly understood action upon the practice of the art.

Magadising, and the crude harmony of the Hucbald type, preceded a real perception of chordal coherence in relation to scalar progression.

Roughly speaking, the recognition of a definite place on the palette of the tone artist precedes the naming of a chord.

Earlier efforts were devoted to the modal aspect.

The continuous pitch flexion possible with the voice; the reduction to a discrete scale by many factors—among which may be included the economy of the fixed pitch elements presented by such early instruments as pan-pipes, flute holes, harp strings, dulcimer plates, etc.—provided a "pitch-linearity" favouring the construction of definite scales, whose actual conformation was ultimately determined by harmonic conditions.

We live in an age of great activity in all branches of science and art, and every field of musical manifestation is being daringly exploited.

One cannot accuse the age of conservatism, excepting, perhaps, as regards certain conventionalities that even the most audacious meekly bow to.

But the present age, besides being enterprising, tends somewhat towards a self-conscious analysis of its doings, and a certain amount of immature statement as to its aims.

The selective process of hyperacoustic determinance grinds, like the Mills of the Gods, slowly; but it none the less surely rejects all that does not come up to its standard of inspiration and fertility.

The process cannot be reversed, and by means of prediction find a royal road to the musical process from a knowledge of

determinants. It is quite as necessary now, as at any time, for genius and hard study to put their whole efforts into actual and finished work before the unknown acoustical criterion slowly moulds the form of classic persistence.

In this process, inhibition plays as great a part as stimulation. Abuse and over-fertility in music are real dangers to be guarded against.

It is fatally easy to fall into primary formulæ, and to stop there; to allow form, melody, tonal beauty, dramatic effect, atmosphere, etc., to usurp the position of master, when they should remain obedient servants.

The present facilities available in musical work peculiarly tend to the diseases of " hyperorchestratis " and " superharmony."

Restraint goes hand in hand with musical energy in the great works. The river must have its banks as well as its hydraulic gradient, or else it will degenerate into a stagnant marsh.

Considerations of a historical nature are far too important to be dealt with in a preface, but the foregoing points have been lightly touched upon in order that the basic principles of investigation may be clearly understood.

These are given in order that the ideas underlying investigation may at least be in the light without prejudice to, or bearing upon, the arguments to be followed.

The two basic principles may be entitled:—

(1) The Principle of Universal Presentation.
(2) The Principle of Genetic Continuity.

The first-named principle posits the reduction of all perceptions and conceptions of, and arising from, external objects, to elementary psychological terms: hypothetically stating the analogy —if not identity—of the elements of hyperacoustics with such terms.

The classification into the " material " and " modal " elements is treated in a section of the present investigation entitled " Physiological Acoustics," but the points are not mentioned now, since they involve questions around which dispute rages.

It is, therefore, advisable to merely mention the principle in order to facilitate the ordering of the concepts which may be evoked by a perusal of the present pages.

Recognition of the extraordinary manifoldity of musical per-

cept and concept (which is as a rule not apparent to, nor generally admitted by the non-musical observer) forms the basis of the first principle.

The external view of music is concerned with the single material medium " Sound."

A similar view of the universe would comprehend the single material medium " Substance." There is, therefore, some excuse for this obsession of monadity on the part of the casual observer, and at the same time, a presumption for the amplification of hyperacoustic theory.

The " monadity " is really implied by the apparent simplicity of the auditory mechanism; a simplicity that vanishes on physiological and acoustical investigation.

Man appreciates the general external world by his multiple sense-organs, from which the conception of physical and metaphysical complexity appears obviously rational.

To the casual observer, the ear is a simple perceptive organ, dealing with a simple vehicle of sense vibrations.

Therefore the idea of any complexity akin to the general concept of externality appears unwarranted by facts, seeming only an exaggeration of petty and purely local conditions.

In the sections dealing with Physical, Physiological, and Psychological Acoustics we have endeavoured to see that the " monadity " or " simplicity " is not a fact: that the aural complex of cerebral, neural, and auditory organisation which we call the " ear," as well as the complex facts which mathematical investigation partly reveals about the periodic variant of magnitude; and finally, that the associated and synergised mental processes concomitant with audition and musical conception, are far from simple.

After a careful study of the Trifold Acoustics, the basis upon which the Principle of Universal Presentation could be maintained is more likely to be admitted by the student. Still more so, if the investigation is carried into the parallel regions of psycho-mathematical extension.

There are persons whose artistic sympathies are awakened by the tangible objectives of Architecture and the natural beauty of scenery, etc., who totally fail to understand how an art which has so little " material " basis as music can move the mind and spirit. Let them be reminded of the difference between what

they perceive, and what many men merely see, in their objects of interest.

This difference arises in the mentality of the perceptor, whose mind is attuned to the significance of the formulation that comes via the senses.

This capability of grasping is not conditioned by the material vehicle of expression. Therefore if it can be shown that there are analogues to the manifold extension of space, the protension of time, and the intension of identity, in the forms of music, there can be no valid reason for denying the Principle of Universal Presentation, whose real justification is to be found in the attitude of the many votaries of the art of music.

After all, the utility of a general conception is due entirely to the suitability of its application at all stages of an investigation.

The Principle of Genetic Continuity directs attention to the ancestry of phenomena, presenting a phylogenetic rather than an anthropomorphic aspect.

Music is not merely a thing of the moment. It stands on the apex of a " pyramid " of reference to past events; and conversely is potentially related to future events.

The logical result of such a principle is to emphasise the continued teleological aspect of the mental processes involved, somewhat akin to biogenetic methods, and it invites parallel comparison with evolution in science and art in general.

The error arising from the drawing of unjustifiable conclusions in terms of purely extraneous conditions has somewhat obscured this particular view. It therefore becomes one of the aims of Hyperacoustic science to place a suitable terminology and method in the hands of the psychologist and sociological historian.

The object of mentioning these " principles " in the preface is to lay open the wide view that must necessarily be taken of the whole subject, as well as to show how the problem differs from what might be surmised upon familiarity with practical conditions.

Hyperacoustics as a science does not seek to supplant any of the empirical studies concerning the practical theory of music. These latter are a part of the general description of a living experience of music, however inadequately expressed from a critical point of view.

Hyperacoustics is occupied with ascertaining what determining

principles persist throughout all ages in conforming practice to certain broad lines of progress; and the preliminary step requires much care in order to abstract the essential from the extraneous.

The study has a psychological value, and, we may add, an educational utility in promoting clear thinking.

Such persistent principles (assuming their reality) require to be extracted from the observations of general practice, which also comprise many limitations due to the extraneous conditions under which musical composition, performance, and audition become possible.

In addition, there are certain less definitive conditions imposed by the gradual development of an empiric art by human beings, themselves admittedly subject to many limitations.

This makes the examination of contemporary work, in particular, a matter of great difficulty, and inevitably introduces the controversial element.

With works of a past age, one can take a more detached and calmer view. Consequently most of the data have to be necessarily drawn from such works, which lays the exposition open to the reproach of not being up to date. The indulgence of the reader must here be craved.

A few additional remarks may throw light upon the particular frame of mind, inevitably personal, in which the succeeding subject-matter is developed.

The simplest comprehensive concept of Hyperacoustics is that of a "Natural Philosophy of Music" in which certain materia are presented as manifestations of individual activity, as works of art.

A musical work premises the capability of composition, possibility of performance, and presumably a sympathetic receptivity (even if confined to one and the same person).

As science can discuss with advantage the materia and modes of organic existence without trenching upon the metaphysics of vital being, so hyperacoustics stands in relation to musical experiences.

The acoustical materia and modes are considered, but the kernel of æsthetics is not touched. That can well be left to the practical exposition of the musician.

The sciences of Natural History, Biology, Physiology, Anatomy, Histology, Cytology, etc., treat of various aspects of the living

being, which appears as an aggregate of matter associated with certain configurations of energy under the control of a predominant unit of individuality.

Such sciences are none the less valuable for the absolute incapability of the human mind to define Life or Personality.

So may it well be with Hyperacoustics, that some gain in knowledge, some aid to appreciation, some stimulus to activity in the practical and theoretical field, may follow, in spite of the fact that music is as indefinable as Life.

The rationale of significance in music is explicable only by definition of music itself as an appeal from mind to mind by means of a special vehicle.

It is a personal act, which involves a manifoldity of personal mentality, in that a musician may stand as sole receptor to his own conceptions.

Throughout the development of music this personality has persisted, but never as an isolated phenomenon, and never indeterminative or as the result of chance.

The logical course of a scientific dissertation based upon comparison of data and experience, requires that if it cannot be shown that experience is capable of statement in terms of data it must be demonstrated why not.

This is the ideal aimed at in the course of the succeeding exposition, and failure to attain the standard does not necessarily involve condemnation of the attempt.

It is sincerely to be hoped that criticism may be based upon the constructive ideal, and that before the tentative scaffold be destroyed, some progress may be made with the permanent edifice.

With that aim in view, it may be pointed out that the arguments, exposition, and language are concerned with the subject as a whole, and should be judged in relation to the whole.

In dealing with any subject in arbitrarily arranged stages, it is inevitable that certain portions should appear weaker than others.

Where data are incorrect, reasoning wrong, or conclusions unjustifiable, criticism is heartily welcomed, but it should not be overlooked that what appears superfluous or gratuitous at any one stage may justify itself in relation to the whole.

To previse any points of critical attack would overload the

whole dissertation until it resembled the legal document armed against all possible chance of misunderstanding and perversion.

In treatises in general, there are two most interesting aspects to be read " between the lines," viz.:—

(1) The underlying philosophy of the writer.

(2) The unconventional and tentative methods which have often preceded exact processes.

Too often, the latter have had to be rigidly suppressed in the interests of the coherent lucidity and conciseness of the work; but it must be confessed that some of the little human touches that occasionally glint through the rigidly logical dissertation are often most illuminating, and particularly stimulating to students of some branch or parallel line of investigation.

This work as a whole is offered not so much as an introduction to the vast subject it deals with, but more as a " conduction," being intended to lie at the left hand of the student when engaged upon standard treatises, with a view to aiding comprehension and stimulating interest in one of the most fascinating subjects presented by nature to man.

With these words the writer commends the result of some years' work to the kind consideration of his readers. Very little that is new will be found within these pages, the plan being principally the collection and arrangement of the thoughts of others in a form enabling the essential continuity of the subject to be perceived.

It is hoped that this book will be read in conjunction with original treatises and investigations on the subject-matter.

In conclusion, the hearty thanks of the writer are due to all those who have aided and facilitated the investigations.

So much kindness has been experienced that adequate acknowledgment is difficult. It can only be hoped that some of the " explorer's joy " felt by the writer may be communicated to his readers.

Every apology is offered for the abstruseness of expression, and a certain amount of recondite expansion, as well as such unavoidable errors that remain in spite of sincere effort, and some considerable amount of revision.

HYPERACOUSTICS

DIVISION I

SIMULTANEOUS TONALITY

CHAPTER I

SUBSECTION 1

"TONE"

THE expression "Tone" may be understood to stand for the sensation evoked by a single musical note, whose physical aspect is obviously a vibration.

Tone is a continuous sensation perceived by the ear, and thus differs from the sensation perceived by other sense-avenues.

To the touch and sight (within certain limits) a vibrating body appears to flutter.

The continuity and smoothness of Tone differentiates it from any other auditory sensation. A general definition of noise is, perhaps, impossible, but one of the main characteristics is the effect of roughness.

A fluctuation of the amplitude of vibration gives rise to a variation in the loudness of sensation, the effect being altogether different to that of tone.

Within a certain range the character of the presentation is one of beating or jarring, so much in contrast to the smoothness of tone that it might almost be called "antitone."

Experiences of this kind point to a peculiar isolation of the phenomenon of tone, whereby it is distinguished from other auditory sensation (indiscriminately classed as noise).

The characteristics of tone render it suitable to be the vehicle of a great art, and it is therefore distinctly marked off from the general phenomena of acoustics.

In studying the science of acoustics, the division specifically concerned with tone (and its contrasting antitone) as employed

in the art of music, may be examined under the name of Tonality, a particular aspect of Hyperacoustics.

The range of physical phenomena of the recurrent or vibrational kind is only limited in frequency by the mechanical conditions.

The extremes of magnitude presented by Astronomy and ultra-atomic physics are included in vibromechanics.

But acousticians, and particularly investigators of Tonality, are only concerned with the limited range of frequencies audible to the human ear; and, still further limiting the extent, attention is principally directed to the sounds useful in practical music, involving limits not only of frequency but also of duration, intensity (loudness), and tonal constitution.

The ear, or rather the whole complex comprising the mechanism of audition, is wonderfully faithful to the concomitant conditions of physical excitation.

If this were otherwise, it is probable that the whole system of musical art could never have attained the stage it has now reached.

Relatively to the vibrational stimulus, the sensation is both established and extinguished rapidly, alterations of attributes quickly and faithfully following the variations of physical excitation. There are limits, which are fully discussed in works treating of the three-fold acoustics. These limits determine the scope of consideration in the subject of Tonality.

It may be shown, by means of the phonograph or other methods, that a tone has four principal attributes, any one of which may be varied continuously and independently of the others. These are Length, Pitch, Loudness, and Character or Tint (corresponding to the physical "attributes" of the concomitant physical vibration, Duration, Frequency, Intensity (measured by the second power of mean amplitude), and Waveform or Vibrational Specification).

Within limits, the length of a tone is equal to the duration of the exciting vibration, the remanent sensation being short in comparison with the periods of vibrations employed in musical practice.

The relation of Pitch to Frequency is somewhat in the nature of a psycho-physical "logarithm."

Loudness appears to increase somewhat faster than physical

intensity. There is no regular method of measurement (although several ingenious proposals have been put forward), but it is possibly a function somewhere between the first and second powers on a numerical basis. The attribute of loudness does not come under particular consideration at present.

The last of the four attributes, Tonal Character or Quality, Timbre, etc., has been shown to depend upon the constitution of the periodic motion, *i.e.* the wave-form.

This is seen to be resolvable into the specification of the Amplitudes and Phases (Vector coefficients) of the simplest components, which may be called "CYCLONS."

The attribute cannot altogether be regarded as a simple sum of sensations, as the above view would appear to imply.

The theory was first presented as a whole by Helmholtz, but the application of the mathematical theorem named after Fourier was due to G. S. Ohm of electrical fame.

We will assume that the reader has some acquaintance with the general theory.

The Cyclon is the tone due to what is known as a simple harmonic vibration.

A particle subject to uniform motion and uniform deviation describes the simplest closed curve, the circle.

Its rectilinear projection upon a straight line is expressible by the "circular functions" of angle, viz. Cosine (along line), Sine (across line), or by the exponential (self-varying) value at its own rate turned through a right angle, or again by the composition of two opposite similar motions.

(For full particulars on these points, see "Physical Acoustics.")

It is possible to approximate closely to pure tone, but it is doubtful if such is ever heard.

Similarly, pure noise (absolutely unmusical sound) must be looked upon as an abstract extreme in the opposite direction.

In practice of musical art, we deal with sounds in which tone predominates; these are capable of being accurately "noted" in pitch, and hence are called Notes.

A Note may be regarded as a sum of cyclons forming a Column or Series. Under usual circumstances only the lowest of these (called the Prime) is evident in pitch, and this nominates the position of the whole group.

The actual vibrational motion is a vector sum of the com-

ponents, the analysis being performed (according to the Helmholtz-Ohm theory) by selective resonance within the ear.

The sounds approximating nearest to pure Cyclons have a somewhat dull, hollow effect, but which is distinctly a "Tint." The effect, which is of a massive character in low pitches, thins to an acute quality upon rising.

When a musical note consists of a number of cyclons, whose frequencies are integral multiples of that of the Prime, the tone-tint (character) of the note varies over the whole range possible.

Description of the relative intensities may be known as Tonal Specification.

The limitation of "blending" to a single note is a somewhat variable function of intensity, but the components are readily noted if slightly differently enunciated, or not remaining strictly alike in all the minute variations of phase, intensity, etc.

This is one of the reasons why it is difficult to build a Note up artificially.

When two or more notes of different pitch are heard simultaneously, various effects arise due to the interaction of acoustical conditions.

Prominent among these is the Antitonal or beating effect, concomitant with the alternate reinforcing and opposition of the vibrational amplitude, a process which is very evident in the plane waves of a liquid surface.

When, however, the group of component tones are in the one particular arrangement of integral multiples of frequency from a common unit, the antitonal effect is a minimum, and the individual cyclons blend or fuse into the general effect of an agreeable continuity.

This arrangement of tones is known as the Harmonic Series.

Certain combinations give particularly suitable "voices" for musical purposes. These have been selected empirically, as the history of musical instruments shows, and brought to a high state of acoustical and artistic efficiency.

The most general theory of acoustics is that which treats of definite magnitudes.

The "sensations" of tone can only be estimated, but it is usual to employ the physical measurements of the vibration.

Frequency represents the number of complete vibrations performed in an arbitrary unit of time—usually the mean solar

second—and is consequently the reciprocal of the period of time required for one complete vibration.

Since definite pitch is the all-essential factor in tonality, it is necessary to express it in numerical form, as based upon frequency.

These matters (which are generally dealt with in elementary treatises) have been discussed somewhat at length, with the view of leading up to the basis upon which a theory of abstract tonality can be founded.

Tone is the most general characteristic of musical manifestation, and forms the material vehicle between mind and mind, primarily by the sense of hearing, secondarily by means of notation.

The very simplicity of this material medium may, perhaps, give rise to a false idea of the subject, but when one contrasts the simplicity of sound with the manifoldity of music, it is evident that an investigation covering both extremes of thought must be highly abstract in character, and hence make some demands upon the patience of the student.

SUBSECTION 2

PITCH, AND ITS MEASUREMENT

The expression, "Pitch" of a tone, may be understood to refer to its position or locus on the range of musical audibility. The term is therefore only valid for frequencies from about 16 to 20,000 per second.

The pitch of a tone relative to another is usually easily perceived and comprehended, but the perception of absolute locus varies widely with different persons, and the circumstances of their musical environment.

Some people are gifted with an almost uncanny sense of exactitude: performers are usually expert estimators with respect to the conditions they are used to, but the average person cannot judge much nearer than half an octave, which is about the extent over which a definite alteration of pitch quality (tonal character) can be noticed.

As a quality of sensation, pitch is not capable of numeric definition, beyond being either "above" or "below" a given reference note.

Frequency is, however, expressible by number, hence the term

"Pitch," whenever used in a determinative sense, has come to mean the particular vibrational measure (or a function of same) corresponding to a particular location of tone.

Frequency is not a pure number, but a count of the vibrations performed in an arbitrary time. It should be noted that French vibrations refer merely to recurrence of position, and not, as with other nations, to recurrence of both position and direction of motion, consequently in such cases the frequencies are doubled.

Since frequency is expressed by a reciprocal of period, the term "periodicity" often replaces it.

The measure of an Interval between two tones is the ratio of their frequencies: this is a pure number, and hence independent of time, and consequently of absolute pitch.

An interval is a transposable entity, since it is unchanged by multiplying its terms by any amount.

The arbitrary unit of time chosen for frequency (fortunately the same throughout the civilised world) is the mean solar second.

A unit frequency thus represents a hypothetical tone of one vibration per second, which may be taken as the basis, and the pitch of any tone can then be represented by the interval between the theoretical unit and the actual tone.

In actual practice, it is not the mere frequency which is required to be noted, but such relations of frequency that most nearly correspond to the sense of pitchspacing on the continuous range of audibility.

As far as can be determined from observation, the sensation of pitch appears to approximately follow the logarithmic formula of the quondam "psycho-physics," first promulgated by E. H. Weber and Gustave Fechner, as commonly holding, *i.e.* the general constancy of ratio between stimulus and sensation.

Each "spacing" sensation steps off afresh in terms of the actual frequency attained.

This relationship appears to hold so truly that the term Pitch may be taken to mean the logarithm of the frequency, with respect to some base to be selected.

Such a logarithm gives a constant difference for the constant ratio of frequencies in an interval.

The logarithmic method necessarily holds when a graphical basis is taken to represent equal intervals by equal magnitudes.

The chromatic scale in notation, and on the keyboard, is a

rough approximation to a table of logarithms whose natural numbers increase in a way that may be inferred from the terminal curve of harp strings, pianoforte wires, organ pipes, etc., due allowance, of course, being made for the other variable factors involved in these cases.

The pitch of a tone can be expressed by $P(n/m)$, where P is an arbitrary constant, and n and m are "frequencies"; the fraction n/m being entirely independent of P.

In this case, pitch is the measure of a number of arbitrary unit intervals reckoned from an arbitrary unit tone.

The unit interval becomes the base of the logarithmic system, and the pitch of the unit tone is zero.

The selection of a unit interval is entirely a matter of choice, but the predominating character of the octave suggests a natural standard, viz. the ratio of frequencies 2 or its reciprocal.

Musical practice divides the Octave into twelve equal intervals known as Equally Tempered Semitones (abbreviated to E.T.S.).

These may be further divided, if required, into Cents, or hundredth parts (A. J. Ellis), or the octave may be divided itself into 1000 parts (Jonquiere.)

The logarithmic system of Briggs is particularly adapted to the ordinary decimal notation, since the integers represent the shift of figures to right or left.

The hyperbolic system of Napier is based upon the exponential function, and has the advantage of representing a number which grows at the rate of itself.

Neither of these systems offers any particular advantages for the representation of pitch, so that the system of logarithms to the base 2 (arbitrarily taken with positive index) is required.

This gives the Binary Logarithm of the ratio of frequencies, which can be calculated from a table of ordinary logarithms by multiplication with the modulus $\frac{1}{\mathrm{Log}_{10}2}$

A few values may be given:—

Frequency.	Binary Logarithm.
1	0.00000
2	1.00000
3	1.58480
4	2.00000
5	2.32194

Frequency.	Binary Logarithm.
6	2.58480
7	2.80734
8	3.00000
9	3.16960
10	3.32194

One of the evident advantages of this system is that the decimal portion (mantissa) of the logarithm represents the locus of any tone within the octave.

By adding to, or diminishing the integer portion, this relative locus becomes raised or lowered in pitch over a corresponding number of octaves, agreeing with the experienced transposability of intervals, and the chords formed of same.

The actual unit of pitch measurement used in practice is higher than the theoretical value given by integral binary powers. The value has risen during the last two centuries, but is still approximated by the theoretical system, which may be conveniently taken as the basis for scientific discussion, "coefficients of correction" being applied where necessary.

The conventional use of spacial direction terms to designate pitch is merely a matter of convenience.

With an increase of frequency, the character of a tone loses its massiveness and tends towards an acute thinness. Moreover, heavy bodies vibrate usually at small frequencies, and, conversely, light and small bodies vibrate quickly, so that the arbitrary convention of "low and high" pitch is not unnatural.

On the other hand, wave-length decreases as pitch rises, and if we followed optical methods by taking as standard, say, a wave-length of one metre in air at 770 mm. and 0° Centigrade, the idea of magnitude would be reversed.

Those used to the keyboard acquire a "chiral" notion of pitch as rising from left to right, while wind and string performers tend to associate high and low pitch with motion towards and from the performer's position.

The idea of high and low pitch is carried out in the semi-graphical system of staff notation.

The abstract idea of "erectness" may be termed Vertication, and will be discussed in due course.

Various systems of pitch notation have been adopted by scientific writers.

That due to Helmholtz is convenient and in general use, but the Binary system enables the number of octaves and their fractions to be expressed as coefficients giving the pitch of a note above the hypothetical unit frequency, at which the interval is unison, *i.e.* Zero (symbol Z).

The table gives the frequency per second, the Helmholtz notation (theoretical pitch, slightly flat with respect to practical values), and the coefficient which equally represents the pitch on the binary system, as well as the Radical Interval from the unit tone of one vibration per second.

TABLE OF PITCH AND FREQUENCY

Frequency.	Letter Notation.	Pitch
1		Zero
2		Unity
–		
16	C_{II}	4
32	C	5
64	C	6
128	c	7
256	c^{I}	8
512	c^{II}	9
1,024	c^{III}	10
2,048	c	11
4,096	c^{V}	12
8,192	c^{VI}	13
16,384	c^{VII}	14

The coefficient 8, representing the " Middle C," is easily memorised.

Integral powers of 2 thus measure the octaves above any assigned tone. Conversely, integral powers of $\frac{1}{2}$ (or negative powers of 2) measure the octaves below.

SUBSECTION 3

" INTERVALS "

Upon considering Intervals with respect to the way in which they are used in musical manifestation, they fall into the two broad groups of suitable and unsuitable.

Unsuitable intervals are near enough to some form of suitable to be comprehended as mistuned presentations.

The suitable intervals are, in just intonation, those whose frequency ratios approximate to small whole numbers (when multiplied by powers of 2).

They are capable of a slight flexibility in tuning, although the most satisfactory effect is obtained with just intonation.

By a slight variation of pitch, such intervals can be tempered so as to contain whole-number multiples of the E.T. Semitone. This is the economic basis of the system of notation in general use.

Employable intervals are classed as:—

(1) Concordant, comprising the Octave, Major and Minor Sixths, Perfect Fifth and Fourth, Major and Minor Thirds, to which any number of octaves may be added.
(2) Discordant, comprising the Major and Minor Sevenths, the Tritone (half-octave), and the Major and Minor Seconds, plus any number of octaves.

The term "discord" is frequently applied in the general sense as embracing the unsuitable "mistuned" intervals. In the present case, the meaning is restricted to the intervals employable in musical manifestation.

It is the characteristics exhibited by the intervals themselves, rather than any conventional method of classification, that is under observation. Accordingly, three general classes may be noted:—

(1) Coherent, or Elements of Chords.
(2) Adherent, or Elements of Scales.
(3) Liminal, or Elements of Temperamental Exchange.

These correspond roughly with the practical classification into Concordant, Discordant, and Enharmonic, respectively.

There appears to be no hard and fast division between the classes.

Each merges continuously into its neighbour through a region of indeterminance; the boundaries being approximately situate at the ratios of consecutive frequencies 6:7 and 24:25.

Intervals of the Coherent class appear as elements of the tone-tint of a chord.

Those of the Adherent class appear as elements of scalar

"flow," and the Liminal class comprises elements of substitution or exchange over a threshold.

Intervals belonging respectively to the three classes may be named Chromes, Fluents, and Limina.

The Radical Intervals between any given note and the tone of unit frequency may be considered as of the Zero Degree of Tonal Determinance, since no simpler relationship is possible.

The binary system provides a convenient method by which the " Grade of Approximation " may be represented, particularly applicable to small intervals.

This consists in finding the number of octaves below the given tones, at which occurs the common prime of the nearest interval formed by two frequencies differing by unity.

Let the Interval Frequency be m/n.

Then the binary logarithm of either $\frac{m}{m-n}$ or $\frac{n}{m-n}$ will measure the Grade.

In the case of small intervals, only one of these values need be calculated, and the result approximates very nearly to exact values.

Example: Required the grade of approximation of the interval 137 : 141.

$$2^x = \frac{137}{141-137} = \frac{137}{4} = 34{\cdot}25$$

With a table of ordinary logarithms, we obtain:—

$$x = \frac{\text{Log}_{10}\,34{\cdot}25}{{\cdot}30103} = \frac{1{\cdot}5346606}{{\cdot}30103}$$

$x = 5.104$, about 5 octaves and one semitone.

By also calculating the value for $\frac{141}{141-137}$ and taking the mean of results, greater accuracy is obtainable.

The results may also be obtained graphically by means of the slide-rule.

The applicability and usefulness of the method will be evident during the course of the subsequent pages.

SUBSECTION 4

"TONAL DETERMINANCE"

If one attempts to analyse the basis of the appeal which music makes to the intelligence (as distinct from the emotions) it appears attributable to the characteristic of Determinance as opposed to Chaos or Chance.

To say that music manifests law and order is perhaps incorrect; since dull and unmusical exercises may do the same. Determinance is not mere obedience to a general law, but it is essentially the characteristic upon which the art of the musician and the intelligence of the theorist may meet.

In order to predicate Determinance, an admitted system is necessary. Since this is essentially involved in the present investigation, the critical aspect must be discussed at a later stage.

In order to exhibit determinance, it is requisite that the elements involved shall be nominated, *i.e.* named, in respect to bases of reference, to which they stand in relationship.

Principal among the nominative terms is the Locus, corresponding to the abstract idea of what is usually understood by "Key."

The term, as employed in practice, is liable to be construed somewhat narrowly. It is therefore preferable to regard Locus as the univalent abstract of the characteristics by which the Key, and its place, are recognised.

The material elements of musical manifestation, viz. notes, intervals, chords, scales, etc., are nominated with respect to either one key, or "Matrix," in which they are contained, or "conserved." But at the stage of a process of modulation in which they partake of the character of both keys, they may be said to be Translational.

The system of Equal Temperament enables the infinitely extending aggregate of keys to be more or less faithfully represented by twelve tones, somewhat in the same way as a curve may be approximately plotted on squared paper.

This is effected by means of the liminal substitution of the "just" tone for one very near to it, which facilitates the coupling up of relationships into Enharmonic cycles; of which more anon.

Translation of the Locus of a Matrix (Modulation of Key)

is thus restricted within a Domain; anything outside being regarded differently to the inherent terms and relationships.

The principle of equal temperament is adopted in the present survey; the numerical values of intervals being understood to represent the small region of temperable variability about the just values from which the tones and intervals are named.

The two principal aspects of tonal phenomena may be divided in the abstract into Simultaneous and Successive.

This is purely a matter of convenience, since both are inseparable in actual cases.

Similarly, there are good reasons for commencing an examination of the phenomena of Tonality with the Simultaneous; rather than, as might have been supposed upon historical grounds, the Successive aspect.

The phenomena of Tonality may also be abstracted into two classes: Chordance and Fluence.

The first of these is concerned with intervals, chords, and developments therefrom; and the second, with the flux of continuity from one tone to another, as in Scales and Sequences, etc.

Objectively, Coherence in chords is simply the superposition of vibrational motions (*i.e.* vector summation of periodics), while Adherence in scales is nothing but the discontinuance of one sound, and initiation of another.

The binding element is beyond the scope of Physical Acoustics. This is one of the reasons why the name "Hyperacoustics" is selected to embrace consideration of the abstracts of Tonality, Rhythm, Organisation, etc., manifested in music.

Chordance may be said to cover the "vertical" aspect of Tonality: and the term Phonality may be proposed to designate the "horizontal" aspect which comes particularly under consideration in "Successive Tonality."

An arbitrary distinction may now be drawn between Concordance—Discordance, and Consonance—Dissonance.

It is convenient to restrict the former to chordance conditions, and the latter (as extremes of a range of characteristics known as Sonance) to the successive aspect.

This is purely a matter of distinction, and is justified by the precision it confers on easily recollected terms.

Standing in contrast to the smoothness and general "agreeability" of Tone is the irritation of Antitone, or the Beating,

Fluctuation, Fluttering, etc., which the old theorist Hollander quaintly describes as a " battel in the ayre."

The theories of Helmholtz on this subject should be familiar to the reader; but a clear distinction must be drawn between Euphony and Cacophony as perceived by the non-musician, and the pregnant tonal aspect in which Concordance and Discordance, Consonance and Dissonance, appeal to the musician.

Space cannot here be spared to discuss these vast problems, about which a considerable literature has accrued.

The process by which the ear accepts tone is debatable, but there are good reasons for regarding the actual mode of operation as following the general theory of harmonic analysis promulgated by Ohm and developed by Helmholtz.

According to this theory, the single " notes " heard are really sums of sensations due to " cyclons " of the Fourier Series of frequencies, *i.e.* integral multiples of the Prime Frequency whose locus is noted. The elementary sensations themselves are excited by the abnormal response due to resonance on a connected range of tuned " syntonisers."

Each element of this range communicates with the cerebral region by an independent nerve fibre.

If this theory be true, it follows from the principles of damped resonance which must obtain in the auditory mechanism, that the cyclon excitation does not merely affect a single nerve terminal, but must influence a small tract concentrated about the locus of maximum syntony; although this " parasyntony " must fall off rapidly on each side of the nominant of pitch, and becomes indiscernable in the general substratum of ever-present excitation.

Since each nerve element sends its own " local-sign " sensation to the brain, whether excited by syntony or parasyntony, it follows that the sensation of tone and the localisation of pitch must be due to the *sum* of these independent effects, or rather, to their outstanding differentiation in magnitude from the general excitation of the whole range.

This is the theory of Ottokar Hostinsky and Alfred M. Mayer.

It may be conjectured, from consideration of the effect of high-pitched tones, that if it were possible to stimulate an isolated nerve element in the auditory region, the effect would be distressingly poignant.

Although the manifoldity of tone is an acoustical fact, it is not directly apparent to the senses, and hence has had to be discovered by scientific investigation, with the attendant opposition usually evoked in such cases.

The two principles, of the Harmonic Series and the Parasyntonic tract, have stood the tests of time and all the experimental resources of modern investigation, although their critics have been many and their views not to be lightly passed by. Nevertheless, the two principles are sufficiently well-grounded to become the fundamental bases of a theory of tonal determinance.

The Harmonic Series exhibits a natural and predominating set of relationships between tones in pitch, to which all intervals may be referred, and in which all the component tones are nominated with respect to Primes.

The Parasyntonic tract permits that small deviation from exact intonation to which is partly due the wonderful economy and generality of the E.T. System.

It also surrounds each cyclon with a region of effect, so that when two tones approach near enough for their "tracts" to sensibly overlap, the effect of beating or Antitone is at once apparent.

Important results spring from this fact, which will be discussed in due course, but the immediate result will be perceived, which is, that each tone spoken of as a sensation is insulated by a region of possible antitone, tending towards the establishment of a more or less definite system of discrete intervals instead of a continuous flexion of tone over the range of audible pitch.

SUBSECTION 5

"THE SERIES"

The measurement of Pitch on the logarithmic system is expressed by the index of the power to which the ratio of frequencies of a base interval is expanded.

The binary system has already been discussed; but it may be mentioned that if the Briggs or Decimal base (which is so useful in arithmetical calculation) was employed, the reference interval would be two octaves plus a major third; while the Napierian or Hyperbolic base is incommensurable, but approximates to an octave plus a fourth.

Neither of these systems presents any particular advantages

from a tonal standpoint, beyond the fact that tables of both are generally available.

The dative conditions of acoustics provide a ready-made system of logarithms, based upon the Octave.

This is the Harmonic Series; the most predominant arrangement of tonal relationships to be found in experience.

Owing to the Fourier principle, "that any periodic function may be regarded as the sum of a series of Sine and Cosine functions (Cyclons), of arbitrary amplitude and phase, but whose frequencies are 1, 2, 3, 4, etc., times that of the Prime" (a condition arrived at when a continuous homogeneous body, such as a string, organ-pipe, longitudinal or torsional rod, etc., vibrates), and the fact emphasised by Rayleigh, "that of all the possible kinds of vibration, the simple harmonic (cyclons) are the only kind which survive unchanged all the physical vicissitudes they may undergo;" the Harmonic Series, out of all possible arrangements, has a preponderance in acoustical, and especially tonal, experience.

The human ear is bombarded with this Series from infancy, and can easily recognise the early members, when trained to pick them out.

Moreover, the conditions of orthogonal forcing, consequent upon the mechanical principles investigated by Faraday, Melde, and others, result in the additional predominance of the first serial interval, the Octave.

Further, the researches of Stumpf into the "synergy" of tone blending (by the statistical development of the psycho-physical method) show that the series intervals possess the capability, to a high degree, of blending into one perceived entity, the grade diminishing in magnitude with convergence in pitch.

In Tonality, we are concerned with the Harmonic Series, and all possible rearrangements of its components.

It is therefore convenient to regard the series in abstract; that is, apart from its actual appearance as an acoustic phenomenon, as a group of intervals.

By taking the Series as given, and its exact converse or mirror image (the series of integral reciprocal frequencies, which we shall have occasion to consider somewhat often), we obtain the two extreme arrangements of the whole possible range of groupings in which the elements are contiguous series intervals.

Generally, this abstract series may be denoted by the terms:—

$$P_1P_2P_3 \ldots P_n$$

where P represents the Prime or unit member, and the suffixes stand for the membership number, or coefficient of frequency: that of the Prime being always unity.

The series measured positively (upwards) may be called Fundamental, the term being applied specifically to the Prime, as the foundation of the Harmonic Column.

The Fundamental Series may be denoted by:—

$$F_1F_2F_3F_4 \ldots F_n$$

Its reciprocal, measured negatively (downwards), may be called the Coincidental, the term being applied to the Note on which an independent harmonic member of each of the Series members coincides.

By analogy, this coinciding note may be known as the Coincidental Prime, and the series written:—

$$C_1C_2C_3C_4 \ldots C_n$$

It may be observed that the series of Notes to whose harmonic components a Helmholtz resonator responds, will be the Coincidental Series of the pitch note of the resonator.

The sum of the Series terms may be put into the form:—

$$\Sigma F = F_1 + F_2 + F_3 + F_4 + \ldots \text{infinity.}$$
$$\Sigma C = C_1 - C_2 - C_3 - C_4 - \ldots \text{infinity.}$$

The symbol "sigma" (Σ) being used in abbreviation for "the sum of" a series.

The general term is $\Sigma \pm P$.

A particular section of a series between, and including, two values may be written:—

$$\Sigma_n^m \pm P = P_n \pm P_{n+1} \pm P_{n+2} \pm P_{n+3} \pm \ldots P_m$$

Since the vector amplitudes (corresponding to intensity) of the cyclon components in the Harmonic Series are arbitrary, it is evident that any desired specification of such a series may be obtained.

By causing all values to vanish except the Prime and one other member, we obtain the "skeleton" series, or Radical Dyad Interval of that member.

This process may be applied to either Series, resulting in forms $(F+F_n)$ and $(C-C_n)$.

The different kinds of Series it is possible to build up are infinite in number. Any definite kind may be called a Species.

We are at present only interested in the two converse extremes, and these Series, or any element of same, may be defined as of the Fundamental or Coincidental Species respectively (F and C).

The characteristic of both extreme species is convergence of pitch upon proceeding outwards from the Prime.

A series of equal intervals is therefore of "neutral species," and may be regarded as the mean between the two extremes.

It may be known as a Homochromal Series, and presents characteristics differing in a marked degree from the two converse converging Series.

It is to be noted that as we proceed outwards in a Series the convergence also converges, so that the "curve" of the members plotted as ordinates, approaches asymptotically, but never reaches, the "straight line" of the Homochromal mean.

It is seen that the Coincidental Species resembles the Phonic System of Von Oettingen (*System of Harmony Dually Developed*, Dorpat, 1864), the "Underklang" Series of Hugo Riemann, and other theories of harmonic conversity.

These are definite attempts to expound an actual conversity of Series, while, in our view, the Coincidental Species is only an abstraction from experience, effected for the convenience of symmetrical comparison and parallel methods of investigation, with the Fundamental Species.

In abstraction, the remainder of the actual characteristics must always be finally taken into consideration, although temporarily excluded for convenience from any particular theoretical investigation.

This remainder, which refers to the aspect of absolute direction established by the Harmonic Series, may be itself abstracted and studied under the name of Vertication.

For the present, however, detailed discussion on this point may be deferred.

CHAPTER II

SUBSECTION 1

" CHROMES "

Owing to certain prominent analogies, more apparent than real, the application of a theory of colour-music (and various ingenious attempts to prove its practicability) has fascinated many minds.

It will be evident, upon a further acquaintance with hyper-acoustic principles, that there is little hope for any progress in this direction. The elements of musical manifestation are extremely complex, and involve many other factors beyond the simple considerations upon which such parallelism is generally based.

The question is, however, a somewhat controversial subject, upon which present discussion is out of place.

It must be admitted that the scientific world is indebted to those investigators who have put their theories to experimental test. Perhaps when man has a colour-voice (*i.e.* can express his emotions by change of colour like a chameleon), some sort of colour-flux sense may evolve.

Without any metaphysical speculations as to a connection between music and colour, it is a fact that the tonal character of a note has frequently been spoken of in terms of " tint," " colour," etc. The fact that this " tint " depends upon the constitution of the Series does not affect the direct perception of character which is comparable with the colour sense.

Following the fusion theory of Professor Carl Stumpf, it is convenient to regard the component *intervals*—not the notes themselves—as the elements of the tone-tint characteristic.

The matter itself is properly considered under the heading of Psychological Acoustics, but will inevitably come under further discussion as the development of the tonal theory opens up fresh aspects.

The Chromal system is here put forward simply as a convenient and easily recalled method of symbolisation for Intervals, which may be called generally Chromes (symbol K).

Whatever form the theory of Chromality develops into at a later stage in the investigation, the reader is asked now to merely regard the colour names as labels for intervals.

Everybody knows the primary and secondary spectral colours; if not, a piece of broken glass held in the light will always provide a spectrum for reference.

A Chrome is to be regarded as an abstract of interval relationship only, and thus independent of the absolute duration, pitch, intensity, and character of its tone components. The term applies generally to simultaneous and (arpeggio aspect) successive presentations of notes.

The term also applies generally to all intervals, but particularly to the five earliest concordant spaces in the Series.

The names apply equally to the justly intoned intervals of early commensural frequencies (Serial) and to the same intervals when mistuned within the limits of tempering.

The three primary colours (on the Helmholtz-Maxwell-Abney basis, at least) are Red, Green, and Violet; which suggests the name " R.G.V. System " for the method.

Allocating these to the three intervals formed by trisection of the Octave, we obtain the idea of the system.

Interval.	Colour Label.	Symbol.
Perfect Fourth	Red	R
Major Third	Green	G
Minor Third	Violet	V

from which, by composition, we obtain:—

Perfect Fifth, G+V	Blue	B
Minor Sixth, R+V	Mauve	M
Major Sixth, R+G	Yellow	Y

and also:—

The Octave, R+G+V	White	W
The Unison, Zero	Black	Z

(The term " Mauve " is used instead of Purple to avoid confusion of the initial letter P standing for the Prime.)

An octave added to any " chrome " does not alter its individual character but, so to speak, " dilutes " it.

Similarly, White added to any tint simply reduces its saturation intensity (within physiological limits).

White added to White gives rise to no new tint. Hence the suitability of the name "White" to denote the Octave.

This peculiar characteristic, manifested both by the interval and the colour which symbolises it, may be known as Achromatism.

Transposition of an interval over any number of Octaves may be known as Achromatic Translation, and consequently the repetition of a given chrome or tone at any number of octaves may be known as the Achrome of the term in question.

The Chromes of the Series between the successive pairs of members are:—

Series Members.	Interval.	Name.	Symbol.
1:2	Octave	White	W
2:3	Fifth	Blue	B
3:4	Fourth	Red	R
4:5	Major Third	Green	G
5:6	Minor Third	Violet	V

and also:—

3:5	Major Sixth	Yellow	Y
5:8	Minor Sixth	Mauve	M

The following terms are applied in their trigonometrical sense.

The Applements of Chromes, with respect to White, are:—

$$W-R=B$$
$$W-G=M$$
$$W-V=Y$$

The Supplements, with respect to Blue, are:—

$$B-G=V$$

The intervals may be summed on the following system:—

Double Violet	2V	The Tritone
Double Green	2G	The Augmented Fifth, M
Double Red	2R	The Augmented Sixth
Etc.		

The Hemichromes, on the same principle, are:—

Semi-octave	W/2	The Tritone
Semi-blue	B/2	"Neutral" Third
Semi-red	R/2	The Hypo-violet
Semi-green	G/2	Neutral Whole Tone
Semi-violet	V/2	Small Whole Tone

The Series Intervals P(6:7) and P(7:8), which are "pseudo-chromes," are seen to lose somewhat of the concordant "chromality," tending towards Fluents or Scale steps.

They are not found in the E.T. System, and may be regarded as Serial Insulators separating the Chordal Coherents and Scalar Adherents.

The two intervals may be respectively distinguished as the Hypo-violet and Super-oscillant respectively.

It may be noticed that the Hypo-violet is approximately one-fifth of an Octave.

The restriction of the term "Chrome" to the concordant "Degree" of interval is based upon the practice of Tonality.

The theoretical principle will be discussed in due course.

Referring to the Logarithmic Binary System of Pitch representation, it is to be noted that the Integer of any numerical expression represents the number of Octaves, and the fraction or decimal (mantissa) stands for the transposable chrome within the octave.

The pitch of any tone may be given with respect to the unit frequency in multiples of W plus the chrome.

Thus, for example, 8W+G gives the middle "E" of the scale.

The enharmonic generality of the E.T. system allows a Chrome to represent any of its "temperable" values.

Thus:—

R represents an "Augmented Third"
G ,, a "Diminished Fourth"
V ,, an "Augmented Second"
M ,, an "Augmented Fifth"
Etc.

Colour Symbolisation may, of course, be carried to a considerable extent, but nothing is gained by complexity.

SUBSECTION 2

"FLUENTS"

A Fluent is an interval which may be regarded as the operator of conversion between one chrome and another. In the passive aspect it becomes a relational expression, appearing as an element of scalar flow (adherence).

In notation, and acoustically, it appears simply as an interval, and therefore is not differentiated from a chrome or any other kind of pitch space between two notes.

The individual nomination as a Fluent is a distinction of tonal determinance, a specific degree of character.

This is the justification for the classification under the particular name.

In the same way that Fluents appear as nominative or operative relationships between successive Series Chromes, so do chromes themselves appear with respect to the Radical Intervals of the Series, and similarly so appear Limina with respect to Fluents.

These relationships may be drawn up in tabular form, in which the Ratios of Frequencies are given by the logarithmic differences of the Intervals.

Series.	Radicals.	Chromes.	Fluents.	Limina.
	K_0	K_1	K_2	K_3
1	1/1			
		W=2/1		
2	2/1		$R=\frac{2}{1}\div\frac{3}{2}=\frac{4}{3}$	
		B=3/2		$V=\frac{4}{3}\div\frac{9}{8}=\frac{32}{27}$
3	3/1		$O=\frac{3}{2}\div\frac{4}{3}=\frac{9}{8}$	
		R=4/3		The " Sharp " $=\frac{9}{8}\div\frac{16}{15}=\frac{135}{128}$
4	4/1		$I=\frac{4}{3}\div\frac{5}{4}=\frac{16}{15}$	
		G=5/4		The Small Diesis $=\frac{16}{15}\div\frac{25}{24}=\frac{128}{125}$
5	5/1		$S=\frac{5}{4}\div\frac{6}{5}=\frac{25}{24}$	
		V=6/5		
6	6/1			

These values, in E.T. Semitones, are:—

Series.	Chromes. K_1	Fluents. K_2	Limina. K_3
1			
	W=12		
2		R=5	
	B=7		V=3
3		O=2	
	R=5		Sharp
4		I=1	
	G=4		Small Diesis
5		S=1	
	V=3		
6			

The Fluents are:—

R Red, in its convertant aspect (the peculiar character of R will be discussed later), 4:3

O Oscillant or Great Whole Tone, 9:8

I Impellent or Great Semitone, 16:15

S Suboscillant or Small Semitone, 25:24

The origin of these names may be noted in the following observations.

Upon successively adding together the E.T. "Whole Tones," we obtain:—

O = Whole Tone, a Discord
2O = Major Third, a Concord
3O = The Tritone, a Discord
4O = Major Sixth, a Concord
5O = Minor Seventh, a Discord
6O = The Octave, a Concord

This characteristic "oscillation" between concord and discord suggests the name Oscillant (symbol O) to replace the somewhat clumsy term "Whole Tone."

(It may be noticed that the colour-symbol Orange, having the same initial, is composed of Yellow plus Red, and thus equals a Major Ninth, the achrome of a Whole Tone.)

The interval between a Mediant and Subdominant, and also between a Leading Note and Tonic, is characteristically one which appears to "impel" the progression. This point will be discussed in detail later, but it will be seen that the term

Impellent (symbol I) distinguishes the " scalar semitone " by a convenient name and symbol.

The Chromatic Semitone between G and V as the difference of two chromes summing to B, is analogous to the Oscillant as the difference between B and R summing to W.

Hence the name " Suboscillant " may be proposed (with its symbol S) to denote this " fluent."

The general name " Antinominant " (symbol A) may be proposed for the E.T. semitone, since the tones of this interval give rise to the greatest discordance when sounded together, and the " keys," from the aspect of signature, are furthest from the original nominant.

It is inadvisable to extend the colour symbolisation to fluents. Certainly, we might use the analogy of agencies converting colours from one to another, by any well-known optical, chemical, thermal, etc., processes, to represent the particular fluents, *e.g.* Acid-Alkali, Hot-Cold, Hæmoglobin-Chlorophyll, Spectral Left-Right, etc., etc., but the operation of colour conversion has no direct and independent sense significance to the human eye.

The Applements of the Fluents to the octave:—

W−O=Minor Seventh, symbol, ultra-oscillant, UO
W−A=Major Seventh, ,, ultra-antinominant, UA

It is seen that B−A=R+A=W/2, the neutral first-order chrome or Semi-octave.

There are two intonations of Oscillant given by the primary relationships in just intonation, viz.:—

$$Y-B=R-V=10/9, \text{ factors } 5, 3, 2$$
$$B-R=9/8, \text{ factors } 3, 2$$

The converting ratio is known as the Comma, 81 : 80, which vanishes in E.T.

These two oscillants are mutually complementary in the Chrome G.

The two similar Antinominant values (distinguished in staff notation as scalar and chromatic respectively) are given by:—

$$M-B=R-G=I=16/15, \text{ factors } 5, 3, 2$$
$$Y-M=G-V=S=25/24, \text{ factors } 5, 5, 3, 2$$

Their convertant ratio is the Small Diesis, which also vanishes in E.T. but is evident in Solfa naming.

SUBSECTION 3

"LIMINA"

The intervals given by the third serial differences (third frequency ratios) may be known as Limina (symbol K_3).

The word "Limen" means "a threshold." It is used because these intervals may be regarded as the convertants by which Fluents (and consequently Chromes and Radical Dyads) may be "tempered" or enharmonically exchanged for each other, in certain systems of economic intonation, of which the most general is the E.T. Dodecanal.

Before proceeding to a detailed discussion on Limina, it is advantageous to consider the general relationship of intervals.

The successive differences between a number of terms, A, B, C, D, E, etc., are given by:—

$$A, \quad A-B, \quad A-2B+C, \quad A-3B+3C-D, \quad \text{etc.,}$$

or generally by the binomial coefficients

$$A - nB + \frac{\lfloor n-1}{\lfloor 2} C - \frac{\lfloor n \;\; 2}{\lfloor 3} D +, \text{ etc.}$$

The successive ratios are:—

$$A, \frac{A}{B}, \frac{AC}{B^2}, \frac{AC^3}{B^3D}, \frac{AC^6E}{B^4D^4}$$

or generally:—

$$\frac{A \times C^{\frac{\lfloor n-1}{\lfloor 2}} \times E^{\frac{\lfloor n-3}{\lfloor 4}} \times G^{\frac{\lfloor n-5}{\lfloor 6}} \ldots}{B^n \times D^{\frac{\lfloor n-2}{\lfloor 3}} \times F^{\frac{\lfloor n-4}{\lfloor 5}} \ldots}$$

so that the number of independent terms of frequencies, or factors of pitch, gives the "Degree" of Tonal Determinance.

Limina are represented by the third differences of the Radical Intervals between a Series member and its prime.

Taking the Radicals from 1 : 1 to 1 : 6, we obtain three values (see the table previously given):—

(1) V—32 : 27.
(2) The "Sharp" 135 : 128, which is nearly equal to an E.T.A.
(3) The "Small Diesis" 128 : 125, which is about 4/10 of an E.T.A.

Two observations may be made on this point.

Firstly, as to the " V " being of chromal dimensions.

The same arguments apply here as in the case in which R is regarded as a Fluent. V(32 : 27) differs from V(6 : 5), the latter being the " theoretical " chrome.

Secondly: The Sharp 135 : 128 is actually larger than the Suboscillant 25 : 24, which is an admitted Fluent. (The actual ratio between the two is 81 : 80, a small interval which is known as a " Comma.")

The application of the principle of enharmonic exchange is, however, confined to the case in which the E.T. notational scalar values are not altered; viz. to the values less than the Impellent 16 : 15, which is the smallest regular scale step.

Hence the Small Diesis, .41059 of an E.T.A., may be regarded as the typical value of a Limen.

It is evident that in the E.T. system, any enharmonic interval is necessarily less than half a step in order to be intrinsically determinate (we are not now considering cases in which determinance is afforded by environment or other conditions).

The Small Diesis is less than an E.T. " Quarter-tone," and thus fulfils the condition.

Moreover, it is the final " difference " of the three " indivisible " chromal elements in the octave, R, G, and V.

The application of the Limen to what is known as temperament, is due to the possibility of slightly altering intervals in chords in order to enable the tones to be exchanged for others near in pitch without notational motion.

In practice a slight variation in actual pitch is often possible, and when used, gives rise to minute shades of effect which can be considerably developed.

Modern harmonic practice appears to be extending this branch of manifestation.

The principle of Equal Temperament consists in the reduction of disposable intervals to conveniently small integral multiples of an arbitrary unit.

Of the many systems that could be adopted, that of twelve to the octave is the minimum limit when the chromes are restricted to the first six of the Series.

Thus the Dodecanal system has been devised and adopted, or rather, has been somewhat reluctantly accepted as the least of the evils consequent upon the necessity of mistuning chromes at all.

It combines the greatest generality of chordal exchange with the considerable economy of only having to deal with twelve units in notation and upon keyed instruments.

This view, of course, does not necessarily extend to the melodic or "phonal" view of the case, which will be discussed later.

There has been, and still is, an immense amount of controversy upon the subject of temperament, which has inspired a considerable literature dealing with the subject in all its bearings.

The rational view is surely that while agreeing upon the advantages which Just Intonation confers upon harmonic sonority and chrome "tint," the restriction to E.T. methods has not prevented the great composers of the past from expressing their musical ideas.

In practice, it is possible with continuously variable sources of tone, such as the voice, strings, sliding brass (and to a limited extent, by "faking" with many other instruments), to attain just intonation, within limits, in performance; and although numerous special notations have been devised, it is generally found that real musicians can be trusted to attune their intervals to the tonal circumstances of the case, rather than by slavishly following the representative E.T. values as noted in black and white.

The more or less complicated methods of attaining or approximating to just intonation upon keyboard instruments have never received general acceptance, but there is an undoubted field before mechanical players, in which the "template" notation of any kind can be applied to the somewhat complex mechanism involved without the difficulties encountered by human players.

It will be observed that the actual mistuning of B and R is slight, but that of G, V, Y, and M is distinctly appreciable.

Some of the approximate equations attainable by means of Equal Temperament may be given:—

$$7W=12B$$
$$5W=12R$$
$$W=3G$$
$$W=4V$$
$$7G=4B$$
$$5V=3R$$
$$8R=3W+G=10G$$

The cycle of keys may be closed by making:—

$$6B=6R+W$$
$$3G=4V$$
$$6O=12A$$

Intervals containing the Series members 7, 11, 13, 17, etc., are not well represented by E.T. values.

Therewith is concerned an important principle to be considered in due course.

SUBSECTION 4

SYSTEM OF EMPIRIC DEGREES

The principle of successive interval differences, corresponding to successive ratios of frequency, provides a logical method of distinguishing a series of Degrees of Tonal Relationship; and as such, appears purely academic, since each Degree objectively presents the same form, *i.e.* the Dyad of Tones.

The method provides a mathematical system of expression and a set of "dimensions," but it may well be asked upon what rational basis such a view is introduced.

For the present it is perhaps too much to expect that the intrinsic and entitical difference between Radicals, Chromes, Fluents, and Limina, shall be comprehended symbolically (in abstract) even if they have been felt and understood as real and separable by the musical perception.

The desired abstraction will possibly follow on further perusal of the developments resulting from the carrying out of the schema, when it is to be hoped that any preliminary impression that such "Degrees" are the result of juggling with terms and numerals will give place to a rational judgment as to justification.

Be that as it may, it is advisable to now turn to actual experience to see if any such distinctions are manifested in tonal relations, and if so, to ascertain to what extent they conform to the theoretical system.

Tonal investigations necessitate consideration of absolutely pure tones (cyclons), and consequently pure intervals.

There is great difficulty in experimentally purging a tone from its harmonics, especially its octaves.

Even if such pure cyclons are generated in air, it is questionable whether they reach the inner ear alone, since they travel by a

complicated and asymmetrical chain of gaseous, solid, and liquid conductors.

Under such circumstances, it is probable that physiological harmonics arise, and we have yet to learn how a cyclon is perceived psychologically.

By various means it has been found possible to eliminate the harmonics to a great extent. Special generators have been used, and also distorted series in which the prime is separated from its nearest harmonics by a wide interval. If a cyclon strikes the ear at an angle, the orthogonal component of the wave train tends to evoke an octave (Faraday-Melde effect).

The cyclon can be approximated, if never actually attained, so that the Abstract Tone considered in these pages may be understood as referring to the common characteristic of the cyclon within the range of tonal audition; somewhat in the way that physicists talk of the absolute zero of temperature.

The methods of musical practice, and the fact that a general "theory of empirical harmony" is not only possible but very useful, prove that the above view is neither extreme nor unjustifiable.

The absolute locatability of the cyclon on the pitch range may be attributed principally to the recognised change of its remanent tone-tint.

This varies, with rise of pitch, from a massive towards a more acute character.

On this basis, theories, such as the "Hell und Dumpf" hypothesis of E. Mach, etc., have been put forward; the idea being that the pitch locus of a cyclon presentation is attributable to the relative proportions of "Lightness and Darkness" in its tone-tint (and in spite of the remarks of Carl Stumpf upon the introduction of a converse duality into a recognised entity, the hypothesis describes the "sensation" as well as any other).

This may be compared with the calculatable and observed convergence of its parasyntonic tract, as noted by Helmholtz, Hostinsky, Mayer, and others.

The change of tone character in a note or compound harmonic "klang," according to the addition of series members, has been attributed by Stumpf to the "lightening" effect of the more shrill partials: *i.e.* to a relative uplifting of the "mass sensation" due to the summed remanent cyclon characteristics.

With regard to intervals, these may be conveniently examined by fixing one tone as "Axis," and widening the interval continuously therefrom by moving the other tone outwards from unison.

A slight departure from unison is at once perceived by the antitonal effect, viz. "beating." The range of effect is differently estimated by observers; Helmholtz puts its range as between the limits 2 and 132 per second, the maximum "antitone" for the middle of the scale being about 22 per second. Wundt practically doubles these values; and recently, electrical experiments with telephones (devised for the purpose of testing condensers and inductances) have given 6 beats per second as the maximum of "hammer."

The most reliable information is no doubt obtainable by the method of weighted "registers" adopted by Stumpf from Weber and Fechner, but even here great care has to be taken to make sure that observers are noting the same things.

It is not until the interval has widened to perhaps two commas (Comma frequency ratio, 81 : 80) that the duality of tone becomes perceptible. This interval may be called Bosanquet's Limen.

In the neighbourhood of this value the second tone appears as a mistuned attempt at unison with the Axis.

The effect is not uncommon in bad ensemble singing and playing.

The region extending possibly from unison to the Diesis—just short of half a semitone—is distinctly liminal in effect, and thus capable of being classed by itself.

Beyond this there is a region in which the Antitonal effect increases to a marked extent, so that the interval may be truly classed as an empirical discord, and named an Antinominant.

Such intervals are sufficiently far apart to be capable of dividing up the pitch continuum into discrete steps forming Scales, while yet not so divergent as to be confused with the arpeggio extension of a concordant chord.

This region is also capable of being classed by itself.

Upon attaining the approximate dimensions of V, the minimum chrome, another region, or class of interval, is encountered.

This is the Coherent, or chord-forming class of Concordants, which stands by itself.

The region is marked by the weakness of the cyclon beats.

All these regions appear to converge with rise of pitch.

Between each class is a nebulous region of characteristic ambiguity, which may be termed " Intercordant."

That intervening between the Limen and the Fluent is hardly perceptible in consequence of the many different justly intoned values possible; fluents having from eight values.

The region of the Super-fluent, about P(8:7), and the Hypochrome, P(6:7), is distinctly ambiguous, and this indefinity, as distinct from clear chromality and fluence, is of considerable importance in the system of tonality, as " insulating " these two Degrees determinately from each other.

A liminal region surrounds each definite chrome, whereby V, G, and R merge continuously into each other, and such theoretical hemichromes as B/2 and Y/2 are not individually evident.

The same considerations apply to B, M, and Y, but there is a distinctly " Antinominantal " interval between R and B, whose centre is occupied by the Semi-octave W/2, which appears as an Augmented R, or Diminished B, and is known generally as the Tritone of three Oscillants.

In the region beyond Y extending to W, the Ultra-fluent (Applemental) region recurs; and about every Achrome, the liminal region of the Nominant Tone is repeated.

From these observations, it is evident that there is a real, although somewhat nebulous, distinction between the different kinds of intervals, roughly corresponding to that derived from the Principle of Successive Differences.

CHAPTER III

SUBSECTION 1

"DEGREES OF TONAL DETERMINANCE"

The idea of a definite "key" in Tonality appears to rest on a set of determinate inter-relationships between tones, constituting as a whole an internally balanced group, which may be known as a Matrix.

This group has a locus on the range of pitch, and is translatable within the domain of the greater matrix afforded by the E.T. system.

In practice, the locus is usually referred to one note, known as the Tonic. This method is eminently satisfactory from an harmonic point of view, but the theoretical "centre of gravity" is not necessarily identical with the so-called Tonic.

What practical musicians recognise is the fact that the locus refers to the whole group, and is not conferred upon any arbitrary member representing position for convenience. On this point the idea is quite in conformity with the theory of Tonal Determinance now being discussed.

The determinance of inter-relationships is effected by applying the conditions of actual acoustics to the existent relations of form; in other words, by selecting from possibilities.

Neither process gives determinate results by itself, although glimpses of part of the truth are afforded by each independent line of investigation.

The pre-existing formulative process may be compared to the physical conditions which determine life, and the selective operation is somewhat comparable to the race factors which settle the actual genus and species.

The method to be followed is two-fold.

Firstly, it is necessary to set out and examine the relationships of purely formal data.

These are infinite, consequently attention must be restricted to the predominant and simple cases.

Secondly, the criteria of acoustical conditions must be applied to the formal theorems.

The aim is the formulation of a "determinant of tonality" which shall set out the basic principles upon which the Matrix, the Domain, and their elements and compounds come to exist and persist.

The acoustical criteria are admittedly nebulous.

The quasi-agreement of deductions from acoustical observations with musical experiences has long been known, and much development has taken place.

On the other hand, by means of formal relationships, in terms of frequencies, etc., it has been possible to prove anything and end with nothing.

Consequently, a combination of the hyperacoustical view, on the one hand, with the principles of economic generality on the other (a sort of principle of Least Action, which will be more explicable as we proceed), is requisite in order to obtain that determinate knowledge which constitutes the data available for a theory of tonality.

A word may be said as to the form in which terms and relationships appear.

These are either terminal or characteristic members of continuous ranges or discrete series.

The leading or extreme members stand as types, while the whole system may merge or converge into indeterminate or nebulous neutrality.

The formal relationships of tonality are ascertained from examination of the coherent, adherent, and inherent structural aggregations.

The acoustic criteria are derived from wide experience in the form of data gathered by observation.

The formal relationships may be stated in the three ultimate modal "attributes" or projections of magnitude known as Degree, Phase, and Order; which are expressions of general application particularly convenient for the examination of tonal determinance.

Degree applies to independent sets of values, each nominated in respect to relationship above and below.

Phase applies to cyclic or recurr nt phenomena, and is primarily familiar in the "sign" of direction.

Order refers to relationships in cascade, which may be regarded as in independent dimensions, analogous to, and illustrated by, the three co-ordinates of space.

The application and rationality of this method will be seen as we proceed. Meanwhile, the reader's kind indulgence must be asked.

There is no natural precedence of Degree over Phase and Order, but it is convenient to take it first, since we have already investigated some of the considerations leading to the aspect.

The general symbol K stands for a Dyad Interval as an entity which may be appreciated and conceived apart from its two tones.

From formal considerations already discussed, we have:—

K_0	Zero Degree	Radical Dyads, $P:P_n$
K_1	First ,,	Serial Chromes, $P_n:P_{n\pm 1}$
K_2	Second ,,	Fluents
K_3	Third ,,	Limina

By means of " successive differences " tables can be drawn up.

We have now to apply the acoustic criterion of the observed " regions " or " tracts " about each nominant tone, and we note the following approximate conditions.

The Externominant, or minimum concordant interval, limits the Series of Chromes to W, B, R, G, V.

The Antinominant, or Interval of maximum discordance, coincides with the series of Fluents from about P(7:8) to P(35:36). At the latter value, antinominance begins to merge into Liminance, the Limen extending from thence to actual coincidence.

In E.T.A. the mean values are:—

Externominants	(Chromes)	5 to 3, say B/2 to V.
Antinominants	(Fluents)	3 to $\frac{1}{4}$, say V to Diesis.
Tempera	(Limina)	$\frac{1}{4}$ to Z, say Diesis to Unison.

These values, in themselves, are non-determinative.

They roughly coincide with the experienced values, but fail to establish what they suggest, viz. a direct chain of reference between the principles of Acoustics and the empirical facts of Harmony.

It is in the application of the inter-dependent and inter-limiting group of Externominants, Antinominants, and Tempera, with their Insulants, to the formal group of successive differences,

that the criterion establishing Chromes, Fluents, and Limina is determined.

Tonality stands or falls as a whole. Incongruous results do not tend to survive as integral parts of art methods.

If the E.T. system is accepted, with its libration of tuning over small intervals, we cannot also accept intervals whose distinction from their neighbours is less than the average amount of equal tempering.

The E.T. system closes the cycles of Domain; permits the free translation of Matrix (key-modulation); reduces the material tones to twelve; enables all intervals to be measured with the same unit; and may be used and abused to an almost unlimited extent.

Its disadvantages have not prevented the great Masters from expressing their thoughts in comprehensible forms, and it enables notation, as well as the mechanism of instruments, to be reduced within reasonably small limits.

Nevertheless, these advantages have to be paid for.

No interval is in tune except the octave; the concordant division of the Series is sharply cut off at P_6, and the further harmonic intervals can only be made use of by special means.

Intervals less than a Semitone cannot be effectively employed, and a general appearance of pseudo-stereotype is conferred upon the system of tonality, which not only affects notation and claviature, but also tends to react upon composer, performer, and auditor.

Such is the dictum of tonal determinance as opposed to chaos and the incongruous results of chance and unintelligent experiment.

Radical Dyads (called for short, Radicals) of Degree K_0 appear as alterations of frequency by serial steps.

Chromes, of Degree K_1, appear as alterations of the radical serial column, *i.e.* as due to the shortening or lengthening of a serial "bundle" component, corresponding to the removal or addition of an element of colour in the tone-tint of the "Klang" or Note ΣP.

Fluents, of Degree K_2, appear as metamorphic or operative elements of flow, by which one chrome is altered into another, while the unmoved terminator (known as the Axis) binds the two chromes together.

Fluents form the basis of tonal scales, as opposed to any other type of scale formed by tracing shapes on the continuous pitch range, a matter which will be considered in the successive aspect of Tonality.

Limina, of Degree K_3, appear as the factors which reduce a real variation of fluence to an axial form; *i.e.* which enable a stationary note to change in name, and by so doing to represent a change of nominance actually equal to a small interval.

Viewed in this light, the Series appears as a statement of terms with converging relationships coming successively under the criteria applicable to their actual dimensions.

The change of Degree is really continuous, *i.e.* Chromes, Fluents, and Limina intermerge; but the intercontrolling determinance of the " cascade of ratios " cuts up the series into sharply defined regions as follows:—

1 : 2	W		
2 : 3	B		
3 : 4	R	Chromal Region	
4 : 5	G		
5 : 6	V		
6 : 7	Hypo V	Insulating Neutral Region	
7 : 8	Super O		
8 : 9	O		
9 : 10			
10 : 11			
11 : 12	Interfluent region		
12 : 13			
13 : 14			Fluent Region
14 : 15			
15 : 16	I		
16 : 17			
17 : 18			
18 : 19			
19 : 20			

The adoption of rigid expressions based upon this " cascade of ratios " by no means involves the theoretical modification of actual conditions.

It is a method whereby the predominating or typical cases of each degree of determinance are set forth in mutual relation-

ship. Each type stands at the head of an infinite range or series.

Some questions may arise upon the "dimensions" of the early formal arrangements.

The difference between the Chromes (W−B) equals R, which thus apparently becomes a fluent comparable with O, I, and S.

We shall have occasion to see that R does actually behave as a fluent in many cases, while it is equally a Chrome.

In fact, this duality is a particular characteristic of the interval, but consideration must be deferred as to this point, as well as on the large "dimensions" of some of the formal "liminal" intervals, until progress has been made with the general theory.

That in any age, the system of tonality in general use is actual perfection, no one can well admit.

It is too evidently a compromise between conditions of what can be done, and requisitions of what is desired; and it is also cramped by many extraneous limitations imposed by practical notation and performance.

But it does represent a stage in an ever-proceeding evolution. It is the nearest approximation to economic generality in the presentation of art works; the growing process being due to the fertility of its exponents rather than the experiments of pure theorists.

In former days, the great musicians carried out magnificent conceptions under limitations which are now disregarded; but when the tyro attempts to overstep the bounds which restrain his eagerness, the weak points are surely revealed by the lack of tonal determinance and the consequent unconvincing poverty of effect, want of conviction, etc.

The physiological question may arise as to whether the sensation of an interval in any way corresponds to a sense of "ratio" between located elements on the syntonic analysis of the ear.

This is outside the scope of the present inquiry, but the synergetic theory as developed by Stumpf and his colleagues would appear to show that the "Chrome-sense" is a definite entity, which has been built up by the active employment of "chromes" in chords, due to the invention of solo-harmony instruments (such as the organ, etc.) which made it possible for single performers to try experiments in effect.

The two aspects which appear in regard to the elements of

any degree are the active or operational, and the passive or nominantal.

By these processes (which have no real continuity outside the mind) expressions are defined, and terms named in relationship to each other.

The property of tones in the relationship of chromes, to fuse into and present a single general effect, has been named Coherence.

It is most evident in the Series, but is also apparent in sections therefrom, and is conferred thus upon chords built up of serial intervals.

Tonal determinance of the First Degree stands for a definite connective property whereby Simultaneous (and, by arpeggial extension, Successive) tones cohere, that is, in the mind of the observer.

Physically, the vector sum of vibrations cannot be so divided, although the wave graphs are characteristic.

The property exhibited by Scales (and, by their extension, into Scalar Sequences, etc.) of presenting the continued linear rise or fall of a single "voice" or "part," which maintains its identity throughout other sounds (the "Phonon" of Successive Tonality), may be called Adherence; for which the Second Degree of tonal determinance stands.

The property of tones which differ but by a small interval in pitch, of inhering in a mutual representative Nominant (the basis of Tempering), is exhibited by the Third Degree of Determinance.

The actual connectives, Coherence, Adherence, and Inherence, are seen to involve something beyond the ordinary conditions of acoustics, viz. interdeterminance within a whole system of tonal relationships, whereby they may be said to be hyperacoustical or quasi-subjective elements.

Between the definite intervals of each degree, there exists a region of indefinity or neutrality, which acts effectively as an insulator separating each adjacent class into groups of nominated dimensions.

These might be taken successively as:—

(1) Hyperchromal, from W to the limits K_1K_0
(2) Hypochromal–Super-fluent between K_2K_1
(3) Subfluent–Super-liminal between K_3K_2

In addition, there are internal dividing regions between and within the intervals of the degrees, viz.:—

Interchromal between B and R as W/2
Interfluent ,, O ,, I as V/2

Careful distinction should be made between the definition of "negative" regions, and any rule which bans them from actual use.

No such implication is of course made in the present examination, which is only concerned with pointing out relative comparisons of determinate character.

It is instructive to set out the values of the Series Chromes upon squared paper representing equally tempered semitones.

Very exact results can be plotted with a good slide rule, and the "Insulation" of the first six chromes is rendered very evident to the eye.

Tables of Series Members corresponding to the Degree of Tonal Determinance

K_0. The Series of Frequencies. Elements of Position.

1 : 2 : 3 : 4 : 5 : 6 7 : 8 : 9 : 10 : 11 : 12 : 13

Elements of Tone Fulness. Elements of Tone Incisiveness.

Each of these terms may be considered to represent an interval (Radical Dyad) from the Unit value:—

$^1/_1 : {}^2/_1 : {}^3/_1 : {}^4/_1 : {}^5/_1 : {}^6/_1$ $^7/_1 : {}^8/_1 : {}^9/_1 : {}^{10}/_1 : {}^{11}/_1 : {}^{12}/_1$

K_1. The Series of Chromes. Elements of Chordance.

Externominantal Type. Antinominantal Type.

$^1/_0 : {}^2/_1 : {}^3/_2 : {}^4/_3 : {}^5/_4 : {}^6/_5$ $^7/_6$ $^8/_7$ $^9/_8 : {}^{10}/_9 : {}^{11}/_{10} : {}^{12}/_{11} : {}^{13}/_{12} : {}^{14}/_{13} : {}^{15}/_{14}$

Concordant. Intercordant. Discordant.

K_2. The Series of Fluents. Elements of Scale.

Externominants.	Antinominants.	Limina.
$^1/_1 : {}^4/_3$	$^9/_8 : {}^{16}/_{15} : {}^{25}/_{24}$	$^{36}/_{35} : {}^{49}/_{48} : {}^{81}/_{80}$
Leaps.	Steps.	Exchanges.

K_3. The Series of Limina. Elements of Tempering.

Discrete.		Quasi-continuous.
$^3/_1 : {}^{32}/_{27}$	$^{135}/_{128}$	$^{384}/_{375}$ $^{875}/_{864}$
Individual.		Approximative.

If we now delete the Octave P(1 : 2) we can obtain a group of limited relations.

K_0	2 : 3 : 4 : 5 : 6	Class Criterion
K_1	$^3/_2 : {^4/_3} : {^5/_4} : {^6/_5}$	E
K_2	$^9/_8 : {^{16}/_{15}} : {^{25}/_{24}}$	A
K_3	$\frac{135}{128} : \frac{384}{375}$	X

The problems arising upon the consideration of tonal structure, as seen in musical manifestation, give rise to numerous questions, of which the following are among some of the earliest.

(1) Why is concordance limited to the components of the Major and Minor Triads, *i.e.* tones of the Series from Members 1 to 6 and multiples thereof, giving intervals W, B, R, G, V, and Y, and M?

(2) Why is the diatonic scale limited to seven members and the chromatic scale to twelve members, and how is it possible to effect so much variety of musical presentation with such apparently scanty material?

(3) Why is it possible to mistune frequencies of powers of 2, 3, and 5, to common pitch multiples, and (although losing somewhat of the harmonious sonority and natural coherence) to perform music with this really inharmonic material in a practically satisfactory manner?

An attempt to answer such questions involves a searching inquiry into the principles of Tonality, which in its turn involves other problems.

The method to be adopted as a preliminary, is evidently precision of definition with regard to tonal terms and relationships. Nothing can be taken for granted in this respect. Following definition, there comes formulation, delimitation, and liminal equation; whereby systems can be aggregated, and checked by reference to actual conditions of acoustics and art manifestation.

SUBSECTION 2

" SERIES ANALYSIS "

The predominance of the Harmonic Series over any other possible arrangement of tones (such as those given by bells, etc.) is due to the fact that this particular series, within a certain

range of proportions, gives rise to the effect of a single "note" of character particularly suitable for musical purposes.

The reason for this suitability does not now concern us: it is a matter for psychological acoustics. But the fact that such series of tones have been consistently selected, and great skill exercised in making instruments and training performers to produce them, is evidence of the experiential basis of their use. The characteristic also extends to the element intervals composing the series, and when man began to utilise the effect of these elements, as apart from (but not excluding) that of notes themselves, the art of modern harmony began.

The chordal conception and treatment of simultaneous sounds, as opposed to the contrapuntal, appears to have arisen about the same time that the extension of solo keyboard instruments made chordal experiment and improvisation possible, and the process was no doubt favoured by the rise of a quasi-graphical system of staff notation, which allowed the chord symbols to be grasped by the eye as recognisable "shapes."

It would appear that organs with rude keyboards have existed for many centuries, but owing to the clumsy touch and lack of pressure balancing (to prevent "wind-robbing") harmonic effects were only obtained with difficulty.

The older letter, tablature, and neumistic notations did not convey to the eye that entity of grouping which the staff notation affords. Everybody recognises how helpful graphical representations are when learning mathematics, and the same principle applies to music.

According to the psychological principle of Synergy, antedating, but first definitely laid down by Carl Stumpf (the name comes from the idea of Specific Energy of the Senses, due to Johannes Mueller), the active employment of a group of associated elements leads to the conception of an entity, somewhat in the way that a compound radical in chemistry may replace an element.

The Chrome-elements in chords were already associated in the Series, and already used in contrapuntal polyphony, as well as some apparently chance cases in which their harmonic character was guessed at.

But they were always recognised as Dyads, *i.e.* two note sounds, and treated as such, until musicians began to think in

and work with the interval effect as a thing in itself. The fact that the process took many years to accomplish; was virulently attacked by the usual well-meaning people; and has at times been abused by over-extravagance; does not vitiate the principle, nor the fact that modern music owes much to the chromal conception and the material thereby provided.

Whether any such process can be observed with regard to Fluents and Limina is a question difficult to answer at present.

The physical preponderance of the Harmonic Series is due to the Fourier principle, as applicable to vibrating systems of uniform dimensions and constitution, such as are eminently suitable for the generation, control, and arrest of musical tones.

The formation of combinational tones by the resonance of asymmetric vibrators, and by the action of beats upon stream motion, is also conformable with the Harmonic Series.

Taken in conjunction with the survival of cyclon vibrations under physical vicissitudes, and the introduction of the super-octave due to right-angle forcing, it is obvious that there is a large number of conditions favouring the Harmonic Series in distinction to any other physical grouping, among which the " squared " series emanating from bars and plates may be noted, with its quasi-octave 25 : 49.

Physiologically, there appears no particular reason for the predominance of the harmonic series, but it is to be noted that chords containing serial intervals will directly or achromatically coincide with any nodal division of the basilar membrane, should such, as suspected by Helmholtz, exist.

The effect of fusion of components into the pitch of the Prime, establishes a sense of absolute direction (Vertication) on the pitch range. Nothing of the sort is to be observed in any other arrangement, particularly in any artificial " Coincidental Series " which might possibly occur in the partials of a bell, etc.

In consequence of the absolute direction of the whole series, a similar character inheres in any of its component intervals and thus the differentiation between Fundamental and Coincidental is the maximum contrast between con- and anti-vertication.

The relative " local-sign " of a chrome in a series is also a directed presentation.

Tonal stimulus can thus be compared to a " touch stimulus " of the auditory nerves, in which a low pitch corresponds to the

pressure of a blunt object, and a high note to the acute effect of a point.

(It should be remembered that in high-pitched notes, some of the partials may fall beyond the superior limit of tone audition.)

To ascertain the critical intensity of harmonic components forming the limit of evanescence involves the student of psychological acoustics in complex problems.

Possibly the organ voicer, who is expert with "mixtures," has the best practical knowledge within his somewhat limited range of disposable sounds. The orchestrator must, of course, have the knowledge, but may not necessarily be able to express it as a Series specification.

The location of chromes within the Series, as apart from pitch, is expressed in terms of convergence.

The Series of Chromes in either Species forms a graded scale of location character; and the Homochrome with its equal intervals contrasts with the former to a maximum extent.

The measurement of intervals in commensural terms requires a unit; and it remains to be seen what value is determined or favoured by the conditions of Tonality.

The Series may also be regarded from the Radical Dyad aspect. This is rational, since the Prime, as unit member, is the locus representative of the whole.

A Radical Dyad $P(1 : n)$ may be looked upon as a chrome of a peculiar type, of larger interval than the Series Chromes $P(n : n \pm 1)$ excepting the Octave, which is the common starting point of all degrees.

Consequently radical chromes are classed in the Zero Degree of determinance, as regards interval character.

But a Radical Dyad may be also looked upon as a Skeleton Series, *i.e.* as a Series from which all the members except one and the prime have been deleted.

The case of the nasal-toned "odd series" is a familiar example of the way in which some of the members can be practically or totally absent.

Carrying the principle of eliminating series members to an extreme (as the pianoforte does on Young's principle with the seventh and ninth members) we eventually obtain the Radical Dyad.

From this aspect a complete series may be built up by a

" Bundle " of such skeleton dyads, standing on a common prime tone.

Analysis of a Series or Chord on this basis may be called " Bundle Analysis," and the idea may be applied, by inversion of arrangement, to the Coincidental Series.

The case is an exemplification, in the special field of Tonality, of a special linear algebra whose properties were discussed by George Boole.

This method of regarding Chords and their component dyads is the essential principle of Thorough Bass, as indicated by the figuration of the components, in which, it will be noted, the numbering is by scale degrees from the bass.

As an example of the conformation of the principle with practical conditions, the following points may be noted:—

(1) The euphony of the " consecutive fourths " in the sequence of first inversions. In this case the component intervals are resolved into sixths and thirds upon bundle analysis.
(2) The admission of the First Inversion of the Triad on the Leading Note into Strict Counterpoint. This chord contains a tritone and should therefore be barred as a discord from a wholly concordant schema, but is resolved upon bundle analysis into Y plus V.
(3) Consecutive fifths between upper parts do not appear in Bundle Analysis, and it is found they do not sound nearly so discordant as when between the bass and an upper part.

The lowest tone of a chord is not necessarily the Prime, but the principle of reference to the bass undoubtedly springs from the Absolute Vertication of the Series, from which the intervals forming chords are derived.

The Fundamental Series is terminated by the Prime, but no such implication can be made with any artificial construction such as the Coincidental Series.

(The " Under-tone " theory held by Riemann and others appears to be due to a non-recognition of the fact that identity must be differentiated from its surroundings. We can no more talk of aliquot vibrations being existent in any frequency, than we can consider the identity of the drops of water in the ocean.)

The possible arrangements of a bundle of independent Radical

Dyads form a range whose extreme species are the F and C Series; being the Harmonic and Contra-harmonic groupings respectively.

These two extreme "Species" possess a large number of characteristics in common. It is therefore convenient to consider them in abstract under one general heading and description, apart from their actual differentiation by Absolute Vertication. This is merely an economic device intended to aid the comprehension of tonal principles.

The formation of Combinational tones of the "Differential" type (which is the most pronounced of this class of phenomena) favours the F Species and therefore Vertication.

Although conditions giving rise to any actual "natural" Coincidental Series are rare, it may be convenient to point out an aspect in which quasi-coincidental chords appear in the Harmonic Series, and in which, by the weakening of other members, they may even become predominant.

A quasi-coincidental triad is presented by F(3 : 7 : 9), but the first actual case is that of F(10 : 12 : 15).

Upon proceeding outwards from this on the series, other combinations, more or less exact, may be found.

In the average harmonic series, the further members are, as a rule, relatively weak, but circumstances may arise under which certain members become pronounced by the elimination of their co-members. Even under these circumstances the Differentials would probably be heard, and since the ear appears to focus the tympanic membrane to sounds of lowest pitch, these would hardly evade notice.

The only case in which quasi-coincidental sounds are likely to be heard would be in the varied series of tones given by solid bodies, bells, etc. Sometimes in the quiet of night, a coincidental effect is suggested, if not actually presented.

When a pulse of tone, accompanied by a general disturbance, strikes a graded series of objects (such as a paling fence) tones lower in pitch may be heard. On the other hand, a forest of trees has been known to reflect a note raised in pitch by an octave, this, according to the investigation of Lord Rayleigh, being due to unequal damping.

If a resonator be held to the ear, and its coincidental series of notes, of frequency $1 : \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ etc., be sounded in compound

notes, the resonator will sing out in response to the component in tune with it.

To sum up these arguments, the Coincidental Series appears as the artificial extreme of a range of chromal reconstruction, whose origin is the natural " blending " Fundamental Series.

The fact that concordance does not depend solely upon the Fundamental Series, but that Minor Chords, apparently selected from the converse, are equally concordant (to the extent of being favoured in olden times?), shows that there is some justification in a view of reciprocal symmetry, even if the matter is not quite so simple as the treatises of Oettingen and Riemann would show; and that Absolute Vertication can be abstracted.

The characteristic of the Prime, in both Species, appears as a binding or unifying element, in which coherence culminates; so that the locus of the Prime represents that of the whole series, however extended.

The use of the Coincidental Series is evidence of the power of Synergy to weld note intervals into chrome entities; for in this Species they are presented in the extreme opposite position to their source, and opposed to all favouring conditions.

The species of a Dyad (two-toned interval) is normally indeterminate.

It is established by the addition of another tone (not an achrome).

Otherwise, the effect of Vertication suggests the Fundamental aspect. This bias is a persistent element in tonal experience, and has some important effects upon chordal development, which will be discussed later.

SUBSECTION 3

" COMMENSURALITY "

Exact commensurality of intervals is only possible when the frequencies can be brought to a common multiple.

Prime numbers are never commensural, therefore tones of the frequency relationship 2 : 3 : 5 : 7 : 11 : 13 : 17, etc., are independent.

Hence there are as many systems of just intonation as there are prime number frequencies involved.

By multiplying each value by some common or commensural

coefficient the products can be made to differ in achromatic pitch by amounts small in relation to the original values.

When this difference falls within the liminal region, the tones can be tempered to either value, or to some intermediate representative.

The values in practical use are multiples of (1 : 2), (1 : 3), and (1 : 5), which can never be exactly in tune with each other.

Two methods are open to adoption when intercommensurality of terms and economy of notes is desired.

(1) To justly intone one particular elementary interval, and to temper the other intervals to it.

The practical use of this method is based upon the grade of chromal audentity.

The octave is such a prominent interval that it reveals any departure from just tuning almost as much as a unison.

Consequently in any system, it becomes almost a *sine qua non* that the octaves shall be just.

Of the other two intervals, one may be either just, or tuned to the octave. In the case of the (1 : 3) intervals the departure from just intonation is but slight; the (1 : 5) intervals are, however, appreciably out of tune, and when tempered to pure fifths or octaves are harsh in simultaneous presentation, although passable in succession.

Pythagorean tuning has the fifths exact: it is adopted to a large extent with stringed instruments.

The distance on the pitch range prevents the octave mistuning from being evident, but the thirds and sixths are decidedly out.

If (1 : 5) be justly intoned, it follows that the fifths and octaves are out of tune, somewhat considerably, as can be seen on calculation or experiment.

The octave divides into three G chromes: one of which can be made exact, the other two being either mistuned, or the note dividing the interval M may be split into two.

This principle is the basis of what is known as Meantone (Mesotonic) Temperament. It is obvious that not only are many of the intervals in a given key upset, but that the modulative capabilities of the key are restricted, or only attainable by means of split notes (*e.g.* A flat, G sharp, etc.) involving a real change of pitch with enharmonic progression.

The error accumulates with departure from the primary

chords of a key; consequently, in former times, a considerable amount of artifice had to be resorted to, in order to avoid the monotony due to a restricted domain. Keyboards and mechanism of complicated types appeared, and the technique of performance was greatly hampered.

Various other methods of tuning, with a view to improvement of intonation, have been devised, descriptions of which may be found in the many works on the subject; that of Helmholtz, with Ellis' appendices, being very comprehensive.

(2) The alternative method is to boldly compromise with the admitted imperfections by distributing the error of mistuning as equally as possible, always keeping the octaves just.

This leads to the merging of sharps and flats by dividing the octave into twelve equal intervals, multiples of which represent the chromes as tempered. There is an undoubted loss of purity in intonation, and the practical labour of tuning is increased somewhat, but there is a great gain of generality in modulation as well as economy of material.

The method cannot be put forward as a perfect solution of the difficulties, since it is simply a compromise whose use has been justified by the effective results attained.

Whenever just intonation is possible, every effort should be made to use it, apart from any pedantic ideas on the subject.

The sense of solid coherence, purity of tone, and depth of sonority, that springs into evidence with a well-trained choir or string quartette, is worth some trouble.

The keyboard difficulty and mechanical complexity militate against the many ingenious attempts that have been made to obtain or approximate just intonation on the organ, harmonium, and pianoforte, but the mechanical player opens a possible field of development in which some of the valuable work done in the seventies and eighties may not be entirely lost.

It has already been mentioned that the grade of approximation with small intervals can be measured by taking the nearest interval between two consecutive serial tones and noting its distance in octaves from the Prime.

The Comma, for instance, ratio 81 : 80, is six octaves and a quarter from the Prime.

The E.T. system divides the Octave into twelve convenient

unit intervals, and chromes can be represented by multiples of these antinominant steps.

The Octave divides into two symmetrical halves, as follows:—

A = Antinominant	7A = B
2A = Oscillant	8A = M
3A = V	9A = Y
4A = G	10A = Ultra Oscillant
5A = R	11A = Ultra Antinominant
6A = W/2	12A = Octave

When the aim of a musical work is directed towards pitch delineation rather than tonal effect, a kind of Fechnerian equal gradation is satisfied by the Antinominant and Oscillant Equal Temperament, presenting the so-called equal hexatonic and chromatic scales.

The application is not modern, nor confined to the western civilisation (see Stumpf's researches into the Siamese and Javanese Scales), but the trend of some modern works shows great possibilities, together with the usual concomitant—opportunities for abuse and exaggeration.

CHAPTER IV

SUBSECTION 1

CHORDANCE

THE chordal system, empirically studied under the name of "Harmony," appears to modern ears almost as an establishment of nature, apart from human activity, the effect being that of exploration rather than innovation.

Investigation into musical history reveals a somewhat opposite aspect, viz. a slow growth accompanied by many divergences into paths now forgotten.

It must not be overlooked that the modern ear is now dinned with musical presentations whose schema is to a great extent chordal.

It is certain that the physical and physiological conditions of audition cannot have changed to any appreciable extent during the comparatively recent period of musical development.

The observations of the Greek theorists, for instance, show that their perceptions were practically identical with those of the present day.

From comparative examination, it would appear that the synergetic processes culminating in chordance, developed *pari passu* with solo keyboard instruments.

The results of the process were not so obvious as to be received without considerable opposition, as evidenced in the history of Monteverde and his contemporaries.

The methods of presentation now at the disposal of the musician have overcome many of the technical difficulties which confronted our forefathers. Consequently, any natural tendencies towards definite chordance can now expand to their full development, a process which is observed to be going on in many directions.

The term "Chord" is usually understood to mean a group of simultaneous tones, or an obvious extension of same in time; as contrasted with the general simultaneous configurations of polyphony. This definition must be widened to include the criterion

of coherence as distinguished from the "self-carrying" simultaneity exhibited by "passing notes," etc.

A chord may be examined as an isolated phenomenon, but it appears in tonality as a stage in a progression.

In general, it is a manifold of all the possibilities of progression, and thus the controversy over names is somewhat beside the mark.

But of all the possibilities within a group, some will be found particularly predominant, and from these the typical names of certain chordal configurations have been taken.

The most obvious characteristic of chordance is its capability of being considered as a range of phenomena whose extremes are con- and dis-cordance.

The differentiation of these extremes (which are only approximately attainable in practice) constitutes what may be termed the "First Principle of Tonality," since it involves all the basic conditions of determinance.

The general system of chords referable to one locus ("Key" ?) may be termed the Matrix.

In the Matrix, any given chord is nominated in relation and with respect to its concomitants.

The number of tone components in a chord enable it to be classed as a Monad, Dyad, Triad, Tetrad, Pentad, Hexad, Heptad, etc.

The inclusion of the first two of these will be explained later.

The number of internal chromes in a chord is $n\left(\frac{n-1}{2}\right)$ of the member tones n.

The number of consecutive chromes, as well as the radicals obtained upon bundle analysis, is $n-1$.

The dyad elements of a chord are those of the coherent, adherent, and approximative degrees.

The first may be regarded as the "static" element, the second as progressional, and the third is the substitutional element in tonal manifestation.

A chord is either a section of a series (in which case it is known as Simple) or a compound of same.

The members may either be all present (Actual) or suggested (Latent), hence the inclusion of the Monad and Dyad as chords.

At present, consideration is restricted to the chord itself. Its

discrimination, by Absolute Vertication or otherwise, will concern us later.

A chord is named from the Species of the series from which it is taken; this gives the two types:—

(1) Serial Chords, divisible into F and C species.

(2) Homochromal, of neutral species, contrasting with both extremes.

The first is convergent, the second is equal; the first is nominated by a Prime, the second has no such natural bias, although it will be seen that Vertication tends to introduce a primal reference.

Monads and Dyads are indifferentiate in species, although the latter are "polarised" by the nomination of their terminators.

The observation that Serial Chords within a certain range, P(1 to 6), are concordant, while Homochromes are discordant, has long been noticed, and many theories have been advanced to account for it.

The reader may be referred to the history of the subject which is treated compendiously by many works on Acoustics and Musical Theory.

A Concord sounds generally agreeable, its elements co-operate towards a general effect, having no implication to any particular progressional direction (achromatic arrangement being for the moment left out of consideration).

A Discord may be agreeable or otherwise, but presents a general lack of co-ordinate unity, blending effect, etc., thus suggesting a composite nature, or an incomplete stage in a progression of tones.

The Greek expressions Syn-phony (Together-sounds) and Dia-phony (Through-sounds) define the perceptions exactly.

The first definite theories on the subject sprang from metaphysical speculations around the relations of numbers derived from the dimensions of concordant sounding bodies.

With the rise of Acoustics as a physical science, frequency of vibration became the basis of criterion, although this view threw a somewhat undue emphasis upon just intonation.

The work of Helmholtz, in collating and extending both experimental and physical investigation in all branches of Acoustics and Audition, developed a theory upon a definite basis resting on the empirics of the sensation-contrast between continuous tone and fluctuating beating (anti-tone) as a criterion of

the purely physical conditions of resonance, overtones, combinationals, etc.

The importance of the theories developed in the great work upon the *Sensations of Tone* can hardly be overestimated, but it should not be overlooked that in other directions much good work had been done.

Just previous to Helmholtz, the philosophically expressed views of Hauptmann had opened the way to a hyperacoustic theory of tonality.

Unfortunately, Hauptmann at the time lacked the acoustical data upon which to develop his theories; and, as his observations were stated in the somewhat cumbrous language of a special philosophy, his views were, to a considerable extent, not understood, and consequently neglected.

The limitation of the wholly concordant portion of the series to P(1 : 6) involves V becoming the minimum chrome.

The series of chromes converge to V, beyond which is a second-order hypochromal region, (6 : 7) and (7 : 8), merging into intervals which can be regarded as definite Fluents. The first of these is the Oscillant (8 : 9), this being the " converting interval " of the first-order chromes B and R.

The chromal differences also converge, also their second and further differences, but still more slowly with every set.

Between G and V comes the limiting fluent S(25 : 24), beyond which occurs a " sub-fluent " region merging into Limina, of the Third Degree.

The convergence can be seen by the approximate values in E.T. units:—

Chromes.	First Difference.	Second.	Third.	Fourth.
W 12				
	R 5			
B 7		V 3		
	O 2		2	
R 5		A 1		1
	I 1		1	
G 4		Z 0		
	S 1			
V 3				

In each of the three separate Degrees there is a nebulous

region about P(6 : 7) at which each tends to merge into the next. This region of indeterminance is capable of somewhat wide interpretation.

If, however, the inter-controlling relationships of the three Degrees are considered together, the tri-co-ordinates determine the boundary at P_6.

Thus O is the convertant operator (fluent) of the relationship B : R and A of R : G and G : V, since, in E.T., both I and S are equally represented by A.

This is one aspect of the "First Principle," and may be termed the criterion of convergence.

It is seen that the sharpness of the boundary between the Con- and Dis-cordant portions of the Series depends upon the preponderance of the Dodecanal E.T. conditions.

If any other system of Octave division beyond twelve (such as those proposed by Huyghens, Mercator, von Janko, Bosanquet, etc.) were used, the sharpness would not be so definite, and further serial members, such as P_7, might be included.

(From the criterion of beating, Helmholtz concluded that the interval 1 : 7 is actually more concordant than M=8 : 5.)

There is, however, another aspect.

The serial member P_7 is decidedly out of tune in the E.T. system. The interval from the Prime lies between (W–O) and the chrome Y, being distinctly nearer the former, and these two values may be called its superior and inferior representatives in E.T.

Taking the nearest of the two values (W–O), we find that the successive serial intervals appear in E.T. as:—

P(1 : 2)	W 12	E.T.A.
(2 : 3)	B 7	
(3 : 4)	R 5	
(4 : 5)	G 4	
(5 : 6)	V 3	
(6 : 7)	(V) 3	
(7 : 8)	(O) 2	
(8 : 9)	(O) 2	

and so forth.

It is obvious that the serial convergence approximates more and more towards Homochromal neutrality or anti-convergence.

P(5 : 6 : 7) appears as a Double Violet, and P(5 : 7) as a Semi-octave, W/2, which, in E.T., is equal to P(7 : 10).

It cannot be maintained that in these cases the convergence becomes actually imperceptible, in fact, it is of some importance in the further developments of the theory, but it is obvious that if the conditions of E.T. are accepted as a working basis, P_6 marks the attainment of a non-serial character, *i.e.* the homochromality or neutral-specified form of chord which stands in direct contrast to the converging triad of either species.

The First Principle may thus be stated in terms of discrimination between Series and Homochrome, the criterion being the E.T.A.

The homochrome constituted of two V or Y chromes is half an octave or its multiple. The interval known as the Tritone bisects W, and the tone P(W/2) is the mean between B and R; it appears as the Arithmetical mean between P(1 : 2) in contradistinction to the Harmonic mean presented by the Serial bisection of W into B and R.

If the Semitone was only an arbitrary unit chosen merely for convenience, the criterion mentioned would lose most of its cogency. But it is a general "average" derived from interchordal conditions, which not only determines the discrimination according to the First Principle, but is also determined thereby.

The E.T. dodecanal division of the octave may be looked upon as an artifice, but it is in conformity with the deepest principles of Tonality, and as a method of musical practice with all that depends thereon, its use has been justified by the results actually attained.

Once this principle is comprehended, the acoustical criteria arrived at by Helmholtz and other theorists are seen in their proper position.

The limiting values for beats, grades of blending (Stumpf), Laryngeal control limits, Intercommensural rhythm (Opelt-Lipps) and other views advanced from time to time, are seen to be but parts of a general principle established upon the basis of Tonal unity and interdeterminance.

This principle in no way limits the extension of Tonality, and does not promulgate crippling "laws" to puzzle students who want to get below the surface of phenomena.

Almost any kind of interval may be used in musical manifesta-

tion, but its definition is determined by the system of tonality in which it appears; and a definite boundary is drawn between Concordant and Discordant configurations by the Convergent versus the Homochromal characteristics.

The limiting chrome V and the limiting fluent A interlimit each other, and the limina below A. They are thus rational determinants depending upon the whole structure of Tonality.

The mistuned "discordances," apart from employable "discords," are not included in this category. They appear simply as distorted cases of their nearest determinate chords.

The Homochromal region about P_6 may also be regarded from the aspect in which it is formed at the meeting-place of two converse series.

When CP differs from FP by one oscillant, the third, and consequently the sixth, members coincide.

$$\begin{array}{l} F_1 : F_2 : F_3 : F_4 : F_5 : F_6 : F_7 \\ \qquad\qquad\qquad\qquad\quad C_7 : C_6 : C_5 : C_4 : C_3 : C_2 : C_1 \end{array}$$

The mutual tone, the sixth member, forms the yoke between two "concomitant" opposed series, in which the common Double Violet chord has a dual valency.

There are other arrangements giving rise to similar overlaps. One of the cases of considerable importance has been pointed out by Hugo Riemann, viz. that in E.T. the values $F_7=C_9$ and conversely $C_7=F_9$; consequently the region of the three tones forming the first serial double oscillant $C(9:8:7)=F(7:8:9)$ when C and F are identical.

Hemichromes, and aliquot divisions of chromes generally, form Homochromes, which are Fechnerian, or equal steps of pitch, *i.e.* equal logarithms of frequency.

These values are quite as capable of delineating shapes and outlines on non-tonal scales, as the formulations of serials.

Since $P(W/2)$ stands midway between P(B) and P(R) and FP(B)=CP(R), and *vice versa* (achromatically), the species neutrality of the bisecting tone is evident.

In relative frequency, the values are PR=40, PB=45. Their harmonic mean is 42.4263 . . . Their pitch mean is 42.5, and $P(W/2)$ is 42.

From the foregoing considerations it will appear that the section of a Series from the Prime up to the sixth member will

have a specific identity, and its members will be differentiated from those beyond the boundary, not, of course, including achromatic recurrents of the first six.

The First Principle of Tonality is thus expressed by the location of a boundary upon the convergent F and C Series.

On the Prime side, the region is wholly concordant; while beyond the barrier, the indivisible terms are nominated as Discordant.

The tones P(7, 9, 11, etc.) will be in a radically different category to those of P(1, 3, and 5).

CHAPTER V

SUBSECTION 1

ORDER

The identity of characteristics presented by the octaves of a tonal manifestation renders the principle of achromatic transposition an important factor in practice.

The effect is most obvious when one tone of a variable interval is held as an axis, while the other passes continuously along the pitch range.

As every octave is attained, there is a sense of return to the axis, so that the effect is wave-like or recurrent.

A slightly similar effect may possibly be apparent with some of the other series chromes, but it is practically negligible in comparison with that of the Octave.

This simplicity of achromatic effect may be attributed to the fact that the Octave is the simplest serial interval to which all others are referable, and there exists no simpler basis to which it can be referred.

It may be compared with the number two in every-day calculation.

Taking any arbitrary tone, its octave is seen to recur at ever-widening spaces on the frequency range; since the values are the natural numbers of equal logarithmic increments.

Frequencies	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Octaves—																		
Primary	1	2	–	4	–	–	–	8	–	–	–	–	–	–	–	16	–	–
Secondary	–	–	3	–	–	6	–	–	–	–	–	12	–	–	–	–	–	–
Tertiary	–	–	–	–	5	–	–	–	–	10	–	–	–	–	–	–	–	–
Quaternary	–	–	–	–	–	–	7	–	–	–	–	–	–	14	–	–	–	–

Each prime-number frequency becomes the Prime of a new independent Series. Similarly, each new recurrent is the prime of a multiple Series.

Analogous schema may be drawn up for B, R, etc.

The selected interval then becomes the base of a particular logarithmic system.

W, being the most predominant recurrent, as well as the standard simplest interval, as the basal terminator of the Series column of chromes, is naturally the unit of reference in the ordinary case. The other chromes can, of course, be utilised if required for special purposes.

The system in which " recurrences " appear, provides a convenient method of condensed representation.

In this, series relationships can be denoted in a compact form by an expression called an " Index of Order."

Order itself is a modal attribute or ultimate expression of term-relation, applicable to any formal development such as obtains in mathematics, for example.

In Tonality, the application has long been realised, but the difficulty has been to abstract and crystallise the perception into precise language.

In this field Moritz Hauptmann was perhaps the pioneer.

The Index was first introduced as a symbol in mathematics to condense and simplify description: it was an economic device and nothing more.

But, as is often the case, the device itself proved a powerful means of extending the comprehension and expression of relationships.

The index system in mathematics gradually grew from a mere numerical symbolisation into a method of expressing operation or its passive prototype, nomination; and from the time of its general adoption may be dated the unprecedented growth of mathematical science, and the technical application of calculation.

The original index was a power system to represent multiplication by the number of terms involved.

Thus, $a \times a \times a$ . . . could be written a^n, where n represented the number of times a was involved.

In a similar manner, other operational symbols, those of trigonometry, the functional calculus, hyperlinear vectors, etc., were devised, and the principles of reversibility, intermediacy, etc., were suggested and adopted.

The interest of the device, as regards Tonality, would be purely academic, were it not that the conditions of actual experience conform to the method and its developments to a considerable extent.

The binary basis of pitch measurement corresponds to a prominent fact of tonal experience, viz. the reproduction of identity at every successive octave from a nominant tone.

The system of indicating the Order of Series Chromes and Tones is by using the numbers representing the integral octave in which such elements first appear, counting outwards from the Prime as zero.

The first octave, P(1 : 2), is undivided.

It is therefore of Zero Order, and its terminal tones likewise.

The second octave, P(2 : 4), is bisected into two Chromes, B and R, by a new tone, P_3, which are of First Order.

The third octave, P(4 : 8), is quadrisected into four Second-Order Chromes, G, V, 7/6, and 8/7, by two new tones, P_5 and P_7, also of Second Order.

(It is noticed that P_5 bisects B; and P_7 bisects R.)

The process may be continued *ad infinitum*, and the Order relationship is seen to express the repetition of a characteristic in " cascade."

The general expression of Binary Order is shown in the following table of frequency ratios obtained by continued bisection of the octave:—

$W = 2$

$B = {}^3/_2$ $\quad$ $R = {}^4/_3$

$G = {}^5/_4$ $\quad$ $V = {}^6/_5$ $\quad$ ${}^7/_6$ $\quad$ ${}^8/_7$

${}^9/_8$ ${}^{10}/_9$ ${}^{11}/_{10}$ ${}^{12}/_{11}$ ${}^{13}/_{12}$ ${}^{14}/_{13}$ ${}^{15}/_{14}$ ${}^{16}/_{15}$

${}^{17}/_{16}$ ${}^{18}/_{17}$ ${}^{19}/_{18}$ ${}^{20}/_{19}$ ${}^{21}/_{20}$ ${}^{22}/_{21}$ ${}^{23}/_{22}$ ${}^{24}/_{23}$ ${}^{25}/_{24}$ ${}^{26}/_{25}$ ${}^{27}/_{26}$ ${}^{28}/_{27}$ ${}^{29}/_{28}$ ${}^{30}/_{29}$ ${}^{31}/_{30}$ ${}^{32}/_{31}$

The result may be put into tonal terms as below, the case applying to the tempered values of each identity within the limits of liminal libration, which are practically determined by the value of the row next below:—

Order.				
Zero	W			
First	B (infra)		R (ultra)	
Second	G (infra-infra)	V (infra-ultra)	Hypo V (ultra-infra)	Super O (ultra-ultra)
Third	Oscillants		Interfluents	Impellents
Fourth	Suboscillants		Tempera	

Similar tables, showing trisection, quintasection, etc., can be drawn up.

Applying the criterion of Order to the "Degrees of Interval," K, groups as follows are obtained:—

One general achrome.

One infra, and one ultra chrome of first order.

Two pairs of second-order chromes.

As regards the latter, each pair is seen to reproduce the relationship of the row above, the first pair being included in the concordant division of the series, while the pseudo-chromes of the second pair fall within an "intercordant" region which insulates the definite chromes from the definite fluents.

The row of the Second Degree comprises the Fluents from Oscillant to Impellent. The central region of this constitutes an Interfluent tract which insulates the extremes of the class.

The row of the Third Degree commences with the Suboscillant (the chromatic or non-scalar semitone) and diminishes to the Liminal Degree. Beyond this point there is no necessity to continue.

Since the rationality of the Fluent depends upon the Chromes, the order of the latter determines that of the former; but, in general, fluents are not designated by Order in consequence of their manifold derivation.

From the aspect of primary occurrence, we may allocate R to Zero, O to First, I and S to Second Order, respectively.

The locus of terms on the bisection table may be shown by means of accents or sloping strokes, if desired.

A stroke downward to the right (\) symbolises infra, while one to the left (/) denotes ultra.

On this basis, simplification of such complex expressions as infra-infra, ultra-infra, etc., can be effected by the use of the conventional symbols:—

K

Infra — Ultra

\ — /

Infra-Infra — Infra-Ultra — Ultra-Infra — Ultra-Ultra

\\ — \/ — /\ — //

Infra-Infra-Infra — Infra-Infra-Ultra . . . etc. . . . — Ultra-Ultra-Infra

\\\ — \\/ — \/\ . . . /\/ — //\

Ultra-Ultra-Ultra

///

There are, of course, plenty of devices suitable for symbolisation of Order relations; such, for instance, as may be extracted from the side and end graphs of a helix, each convolution having a "pitch" of one octave, in which case, first-order elements appear as semicircles; second-order, as quadrants, etc.

On this system, chords can be represented by combination of their elementary forms, and their particular "inversion" also shown.

The ground idea is the representation of the real experience, first insisted upon to any marked extent by Hauptmann, viz. the independency of characteristics exhibited by the three tonal and chromal components of the Triad.

If the octave be divided into twelve "Antinominant" units, and the regions of parasyntony about the Prime and its Diametrical πP, and Octave, be indicated by some such device as shading, a diagram is obtained representative of the "real experiences," con- and dis-cordance, of each of the intervals sounded with the Prime, or its Achrome.

The insulation between Chrome and Fluent, and between Oscillant and Impellent, by means of the neutral hypochromal and interfluent regions respectively, is of importance.

It is observed that the Ultra (right-hand) elements of bisection are of this type.

The Hypo-violet, 7/6, and Super-oscillant, 8/7, insulate the chromes from the fluents.

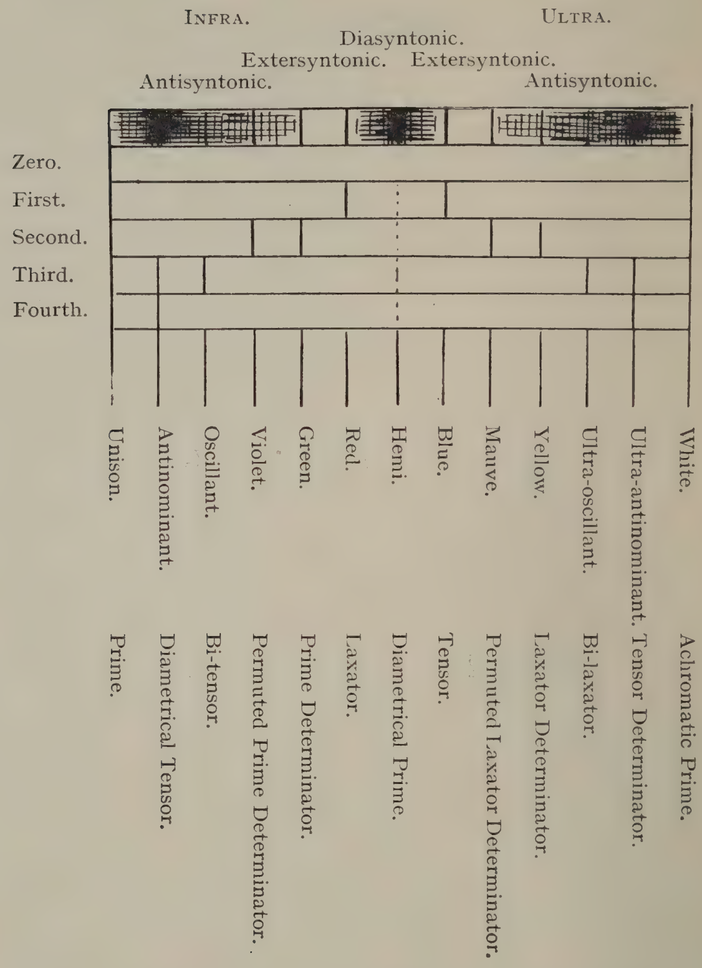
The Interfluent class, 11/10 to 15/14, insulate the two types of fluent.

The Hypo-fluent and Super-liminal region insulates the respective elements between which they are located.

It is perceived that the "indefinite" intervals, which are not utilised directly in a musical system, have a very important function in chordance, due to the characteristic of contrast, in virtue of which they become partitions between definite critical regions.

It is to be noted that R, although experientially to be regarded as a Chrome, is insulative in comparison with B and G, which it separates, and it also acts as a "fluent" or converter between W and B.

We also note the insulation of the Prime from its nearest concordant relations, G and V, by the fluent region surrounding it, which is repeated at the achrome by the insulatory region separating W from Y and M.



It is further seen that W/2 insulates the two first-order chromes B and R.

The insulatory character of these intervals is inseparably bound up with the whole basis of Tonal Determinance, so that one cannot be considered without the other, the contrast of presentations being the most important factor in chordance.

It is noted also that the " Intercordant " P_7 and its analogue P_9 are both in the insulatory region surrounding the octave of the Prime.

In the diagram it is seen that the ultra first-order chrome R and the infra second-order chrome G are apparently not separated by any insulation element.

A similar state of affairs prevails between B and M.

The chromes themselves are of distinctly different order, but they approximate within a region of extreme temperability, and can be deformed by continuous mistuning into each other (*i.e.* a " diminished " fourth is G).

These two regions are those in which the Impellent fluent, 16/15, is the converting ratio.

They are regions of maximum change of order by minimum pitch variation; hence, the Impellent fluent may be expected to present a maximum of operational activity, as compared with the Oscillant and Suboscillant.

The Impellent fluent, as centre of the fluent class of interval, holds a particular position in tonal relationship, which will be evident upon consideration of the circumstances of the name " Leading Note " in empirical harmony.

Taking a succession of tones of Zero, Third, Second, and First Orders achromatically reduced to their nearest position in pitch, we obtain a Quadrinomial which is seen to form a semi-octave scale.

This, in conjunction with a similar arrangement reversed, presents a seven-toned scale.

The orders of the components in the compound arrangement are symmetrical about the tone (not included) which arithmetically bisects the octave.

Reckoned from an axis tone, the succession of chromes G : R : B : Y present the four tones of the scale.

Thus, from the axis Doh, we have Me, Fah, Soh, Lah.

Taking V : R : B : M from the axis tone Me, we have Soh, Lah, Te, Doh.

The possible functions of the further hemichromes B/2 and R/2 are seen to vanish in the general tempering of second-order chromes. Hence these values do not enter into consideration at present.

It may be noted that $P(3 \times 7 \times 7)$ approximately bisects the interval B formed by (P : T).

By such methods, a system of "quarter-tones" (half semitones) can be devised; but as long as the liminal system of E.T. remains the predominant basis of expression, such extensions can only be secondary material in tonal manifestation.

SUBSECTION 2

COMMUTATION AND PARTIALISATION OF SPECIES

The discrimination of Species may be regarded as applying not only to the whole Series, but to any of its elements.

A Series may be divided up into either Radical or Consecutive Dyads.

The former present the aspect of a "Bundle" of Radical Dyads extending from the Prime as a common axis note.

The latter present the transposable entities, Chromes.

The operation of converting one species into another, by any means whatever, may be called Commutation.

The two Species are thus in mutual commute relationship to each other.

A Dyad cannot be actually turned upside down, but the names of its terminating tones can be changed over.

It thus appears dual, by being inherent in both Series.

The commutation of a whole Series can be effected in two ways, viz.:—

(1) By "sliding" all the elements of a "bundle" so that the axis is transferred to the other end.

(2) By rearranging the Series Chromes convergently in the opposite direction. In this case some or all are bodily translated in pitch relative to the Prime.

But this process need not be carried out as a whole. A Series may be resolved into its partials of individual Order, each of which may be commuted separately.

The result of such "Partial Commutation" is a Composite Species.

Partial Commutation of the Zero Order is merely achromatic redistribution of components relative to the pitch locus of the nominant, resulting in what are known as chordal "inversions."

Its effect is to swing round the affected tone from above to below the nominant, or *vice versa*.

Partial Commutation of the First Order involves the trans-

position of first-order chromes, with the consequent displacement of the bisecting tone P_3 over an Oscillant in pitch.

Partial Commutation of the Second Order is applicable within the infra and ultra first-order chromes, and, for distinction, may be known as Permutation.

Thus G and V may be transposed within B, involving a displacement of P_5 over the Fluent $S=\frac{25}{24}$; or the hypo-violet and super-oscillant may be similarly transposed within R, the tone affected being P_7, the Contra-determinator.

Partial Commutation may also be applied between the Orders; *e.g.* R and G may be transposed within Y, the convertant being the Impellent Fluent, etc.

For convenience, consideration is here restricted to the concordant chromes of zero, first, and second orders.

The intercordant regions can be considered later, although it may be now noticed that R splits, like B, into two second-order (pseudo) chromes.

The "Orders" may be regarded as elements which can be compounded in various ways.

For the purpose of illustrating this property in a comprehensible manner, recourse may be had to the imagery of three-dimensioned space.

Of course, no metaphysical implication is involved in the use of this illustration. It is simply owing to the fact that because the three orthogonal dimensions of space afford an evident example of independent composition, the analogy is so aptly used in many fields for illustrative purposes.

The three concordant Orders of Tonality may be pictured as projections upon the three orthogonal dimensions of space, as coexistent yet independent magnitudes.

The Zero Order may be associated with Vertical projection; fundamental octaves being reckoned upward and coincidental downward.

The sign of the frequency index 2^{+1} specifies the particular direction.

The First Order may be regarded as projected in a chiral direction on the same basis F(B and R) as dextral, C(B and R) laeval.

For the representation of the Second Order there is only available the pro-contra direction.

G is regarded as leading from (Educt), while V, the Adduct Chrome, leads back with respect to a given Prime.

The Partial Species of each Order can thus be represented independently, and compounds of the elements can be delineated by spacial co-ordinates.

The personal basis of regarding direction in space is utilised in order to provide an easily remembered method of distinction.

Vertical representation of Zero Order is derived from the conventional idea of the upwardly converging harmonic series.

The allocation of the chiral direction to the First-Order elements is borrowed from the keyboard, while the pro and contra idea of Second Order conforms with the motion of the " Mediant " tone in the close position of a triad.

The space terms used as names are chosen merely for the purpose of bringing vividly to the mind the independent co-existence of the Orders, in conjunction with their direct and converse relationship to the original series.

To persons trained in the mental visualisation of abstract relationships (such as mathematicians), the imagery may savour somewhat of childishness; but even then it must be admitted that a definite and easily recalled name is an economic device of some use.

The two terminal tones of any interval may be known as its Poles.

The idea of polarity, however, is, for convenience of distinction, restricted to the First-Order terms.

CHAPTER VI

SUBSECTION 1

THE TRIAD

When tones are considered in the simultaneous aspect of chords, it is convenient to denote the number of components in the group by the Greek terms: Monad, Dyad, Triad, Tetrad, Pentad, Hexad, Heptad, etc.

On the other hand, when the successive or any other relationship is implied, the Latin terms, Nomial, Binomial, Tri-, Quadri-, Quinta-, Sexa-, Septa-nomial, etc., may be used.

Among chords, the three-toned groups known as Triads hold an important place as the determinately limited aggregate of unity in coherence, species, locus, etc.

The term applies generally to the group obtained by achromatic reduction of the concordant portion of the Series, and also to any three-toned chord of different components.

Owing, however, to the pre-eminent simplicity of the concordant triads (which are also known as Common Chords) it is convenient to restrict the term "Triad" specially to this class, indicating other forms by particular designations.

This is in accordance with accepted practice.

From the two converse series there are obtained two species of three-toned chords, viz.:—

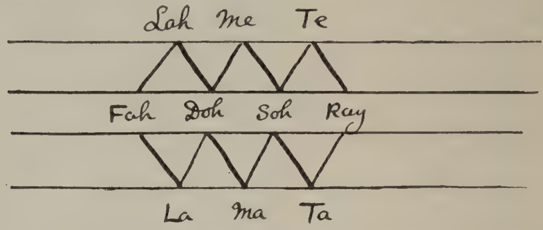
Fundamental Triad	F(1 : 3 : 5)	Symbol	△ (Delta)
Coincidental Triad	C(1 : 3 : 5)	,,	▽ (Nabla)

The general expression for an unspecified triad is P (1 : 3 : 5), and the symbol $\underline{\breve{X}\bar{X}}$.

The triangular symbols, inscribed with the nominant prime, were suggested originally by Andre.

The symbols follow naturally from the arrangement of parallel rows of tones in First-Order relationship horizontally, and the

second tones written vertically over the intermediate spaces, their permutes being subscribed similarly:—



The direction of the Apex thus corresponds to that of the Species.

Every triad is seen to be associated with two of the opposite species.

The F triad is identical with the Major Common Chord, its companion in the upper row is its Relative Minor, and that in the lower row, its Tonic Minor concomitant.

The Minor " Tonic " tone is seen to be the Coincidental Tensor, but no confusion need arise on this point if the abstraction from Vertication be remembered.

Triads can be arranged in three positions, according to whichever component is lowest in pitch.

The bundle analysis of the internal intervals is seen to be:—

Position.	Bass Note.	F Species.	C Species.
Root	P	(B + G)	(B + V)
First Inversion	D	(M + V)	(Y + G)
Second ,,	T	(Y + R)	(M + R)

The Root position is taken as the general basis of consideration P(1 : 5 : 3).

The tones and chromes are of the three independent Orders.

The tonal component of Zero Order is the Prime P, whose " partial species " is identical with that of the Triad.

The tonal component of First Order is P_3, the component which " extends " the Series by appearing as the first non-achromatic element.

It may be called the Tensor, symbol T.

Upon commutation, it becomes the Laxator, symbol L, which is the converse of the Tensor. $L=T^{-1}$, $T=L^{-1}$.

The partial species of the first-order elements may be termed the Polarity. It is distinguished by the chiral distinctions of direction, Dextral for FT and Laeval for FL, the Coincidental values being reversed.

It is to be noted that FL=CT, CL=FT, with respect to a common Prime.

Of the two second-order tones in the Series, P_5 and P_7, we are concerned only with the first, since P_7 does not appear in the concordant Triad.

P_5 is seen to be the tone conventionally known as the Mediant. Since it "determines" the actual species of a triad by one of its bi-valent forms, it may be known as the Determinator, Symbol D.

The partial species of the second-order tones is expressed by a separate variability of the order; the serial form identical with the species of the whole triad may be known as the Educed Determinator D and its "permuted" form D^{-1} as the Adduced or Permuted Determinator **D**.

The intervals of the triads, in close form, are as follows:—

$$P : D : T = G : V$$
$$P : \mathbf{D} : T = V : G$$

D and **D** therefore differ in pitch by a semitone.

The commute variability of Species may be distinguished in the partials applicable to each independent order of component presented as follows:—

Zero Order.—By relative inversion of primes.

First Order.—By reversal of polarity.

Second Order.—By permutation of determinator.

The variability of the First and Second Orders may be seen on consideration of the member harmonically bisecting the next lower Order of interval.

Bisection of the octave may be effected by the interpolation of either T or L (T^{-1}), in which cases we have respectively W=(B+R)=(R+B).

The harmonic bisector may be said to move over one Oscillant Step in effecting reversal of polarity.

Bisection of B (R will be considered later) into two second-order chromes may be effected by the interpolation of either D or **D**, giving rise to the respective cases B=(G+V)=(V+G).

The harmonic bisector (the Determinator) moves over one

Suboscillant (Chromatic Semitone), the tone being permuted from educt to adduct partial species by the operation.

The symbols ▲ ▼ indicate triads whose determinators have been permuted.

The remaining tones of the chord do not move, and thus constitute the axis of the operation.

The second-order variability of M and Y is exactly similar to that of G and V.

Commutation is a directed operation, whose unsigned repetition, like that of the mathematical symbol minus, restores the original state of affairs.

This introduces the directional idea of the operation.

Commutation may be continued by repetition in the same direction, in which case nomial relationships are derived, as follows:—

Zero Order	Sub- and Super-Achromes.
First Order Dextral	Tensor, Bitensor, Tritensor, etc.
,, ,, Laeval	Laxator, Bilaxator, etc.
Second Order	Determinator, Bideterminator, etc.

These processes are translative with respect to Matrical Locus.

It is evident that FT and FL correspond with the Major Dominant and Subdominant respectively; the corresponding Coincidental values are reversed in respect to the Minor Mode.

This may appear to introduce an unwarranted complexity into the consideration, but it must be remembered it is the converse symmetry of the species that forms the basis of our abstract consideration.

We may now turn to the Second-Order tone P_7, which is beyond the concordant region of the Series, and does not appear in the triad.

Since it harmonically bisects R in the same way that the determinator bisects B, the tone may be called the Contradeterminator, symbol Ɑ (for short, the Contra), and its permute, with respect to the original Species of the series, may be symbolised by ◖.

In the E.T. system, which represents a construct of triads, this tone is situate between the Bilaxator (or permuted Tensor Determinator) and the Laxator Determinator, being rather nearer the former.

These two notes therefore represent the educt and adduct forms respectively.

In Tonic-Solfa symbols, we have—for the concomitant values:—

	F Species.	C Species.
P	Doh	Me
T	Soh	Lah
D	Me	Doh
◗	Ma	De
ᗡ	Ta	Fe
◖	Lah	Soh

The characteristic of a triad is the retention of its coherence (apart from physical concordance) under all conditions of achromatic and plural arrangement.

The inversions certainly differ in effect, especially the second inversion, with R in the "bundle," which has a peculiar directive character. This property will be considered later.

The abstract character of a triad does not depend upon the agreeability, or upon the actual amount of "antitonal" beating presented by the sounded chord.

The closer positions of triads are considerably antitonal when low in pitch, and triads whose component notes are rich in harmonics may be distinctly and even painfully rough, but this does not disqualify their claim to be considered as triads, nor mask their recognisable coherence within wide limits.

The three possible vertical arrangements are first found in the series at P(1 : 3 : 5), (3 : 5 : 8), (5 : 8 : 12).

Each component appears in turn as the basal terminator.

By achromatic reduction to the most compact form, we obtain:—

$$P(\tfrac{4}{4} : \tfrac{5}{4} : \tfrac{6}{4})\ (\tfrac{3}{4} : \tfrac{4}{4} : \tfrac{5}{4})\ (\tfrac{5}{4} : \tfrac{6}{4} : \tfrac{8}{4})$$

The elimination of either component renders the chord intrinsically ambiguous in species, but in practice the conditions of environment may suggest the "latent" member.

A triad is nominated and located by its Prime. The term "triad" covers all achromatic extensions, so that it is possible to have "triads" of some forty tones within the pitch range.

The number actually used is determined by the conditions and requirements of practice. In very "thick" part-writing triads of a dozen or so tones may be found; the usual basis

being the tetraphonic or four-part harmony (which is not merely an arbitrary arrangement). In this case one component of the triad is achromatically "doubled."

The effect of a multiplicity of components is somewhat ponderous. Best effects are obtained with an equable distribution of tones slightly converging upwards, and situate at about the centre of the pitch range.

It may be noted that (W + G) is particularly effective in the lower regions of pitch. The Janko keyboard has the advantage in this respect over the ordinary arrangement in enabling the wide intervals to be reached easily.

The Helmholtz criterion of concordance, based upon the relative jarring of overtones which fall within each other's parasyntonic tract, is considered to favour the F triad; but it is to be noticed that if $F(CD)_3$ jars with $F(CP)_5$, FD_3 jars with FP_4, and FD_5 with PT.

The intervals of the F triad, being part of the Harmonic series, naturally blend better than those of the C triad, as shown by the statistics of Stumpf's psycho-physical registers.

The combinational (differential) tones of the F triad concord better than those of the C triad.

Although the triad is far from being a perfect concord, it does represent the nearest approach to freedom from discordance possible with three independent components.

If the tones are cyclons they fall (in all but the lowest employable pitch regions) outside each other's effective parasyntonic tract, so that only the combinational resultants can then affect the purity of the concordance.

Of these, only the first few differentials are obvious, and these are well clear of the generators.

The possibility of tempering permits libration of the components within the liminal regions.

As already seen, the E.T. triad is out of tune, both T and D being tempered with respect to P.

The Pythagorean Triad has T just, and the Mean-tone Triad makes D just, within a group of close relationships.

The extreme limit to which triads may be distorted is determined by the form of the resultant chord remaining distinct from any other triad, and always referable to its origin.

On this basis we have Augmented, Imperfect, and Diminished

Triads, etc., whose function is that they can replace the perfect forms in developing polyphonic sequences and imitations, thus permitting the shape of the chord being translated along a scale or arpeggio.

Certain transitional forms are also included under the heading.

The " pseudo-triad " formed by P : T : ꓷ/◖ (*i.e.* by bisection of R) is seen to follow the same general principles as its concordant prototype.

Its place in tonality will be considered later.

The odd-numbered members of the Series beyond the concordant boundary give rise to what may be called Quasi-triads, which in some harmonic combinations may possibly become evident and prominent.

The approximations of some of the earlier ones may be noticed:—

P(5 : 7 : 9) to the Commute Triad (CP = TT)
(7 : 9 : 11) to the Nominantal Triad (FP = LL)
(9 : 11 : 13) to the Commute Triad (CP = LD)

The series of the Tensor, in particular, approximates somewhat to the tones of the diatonic scale.

Those of the Laxator and Prime are less close.

The interval 7 : 11 is certainly nearer 8 : 5 (M) than to 3 : 2 (B), the actual ratio being 105 : 110 : 112, but its bisection by P_9 tends to favour the recognition of its shape in a Quasi-triad.

It is to be noted that in E.T. CT_7 = FT, and *vice versa*, with respect to the same Prime.

The semi-octave W/2 may be regarded as an Augmented R or a Diminished B. It is, therefore, of neutral polarity. The following relationships hold:—

Augmented B = M
,, O = V
,, G = R, etc.

These are about the most extreme cases of distortion usually met with.

These considerations introduce another aspect of tempering, in which the rationality of the practice is due to a compromise between the phonic and chordant (melodic and harmonic) conditions of presentation.

The rationality of the augmented and diminished triads arises from the use of pattern-imitation. The "shape" of a Triad is reproduced upon other degrees of the scale, such as occur in sequential passages. Such chords, although not strictly concordant triads, possess so many characteristics in common with their prototypes, as to be considered as if they were really triads that had been stressed into a distorted form within the limits of liminal libration.

Investigations have been made to ascertain what form of second-order interval is actually played when the tones occur in succession, and it would appear that violinists and vocalists tend more to the Pythagorean Third of 64 : 81 rather than the pure value 64 : 80.

Tonality embraces all the conditions under which tones can be used in musical manifestation.

The coherent aspect emphasises just intonation; the adherent (scale form) involves a certain amount of accommodation in order to maintain chordal shape; and the inherence of chords in a matrical system of unique nominance, together with the inherence of the matrix in a domain of cyclic transposability, involves compromise of exact pitch.

Hence the E.T. is based upon something more than an arbitrary concession to the exigencies of practice.

The independence of the order components, and corresponding partial species, permits the formation of chords of mixed species.

The varieties possible may be deduced from the number of disposable elements.

In this connection it is obvious that the so-called Major Triad of the Subdominant is of mixed species, for its first-order element the Laxator is the Coincidental tensor of the prime, while its determinator is of fundamental species, being actually the permuted coincidental determinator of the prime. This suggests that the major mode may not be so homogeneous, nor the harmonic minor mode so heterogeneous, as conventional notation would imply.

The triads formulated by P(16 : 20 : 24) and P(16 : 19 : 24) approximate to educt and permuted triads respectively. This point has been noted by theorists, but since the nineteenth harmonic must be exceedingly weak, the aspect is of but little

importance except in the case in which P_{19} appears as a transitional element between P_{18} (TT) and P_{20} (PD). The case might occur when these three harmonics were blown on a narrow tube, such as a trumpet.

The arithmetical bisection of chromes (into homochromes) may now be considered.

In the case of the Octave we have P : PW/2 : WP, which appears as the mean between the F and C " harmonic " bisections P : T : WP and P : L : WP.

Similarly, by the bisection of B we obtain the Symmetrical Triad P(Z : B/2 : B).

The note which replaces the Determinator is ambiguous or neutral in species, having the effect of a mistuned E.T. tone.

For theoretical considerations, the idea of such a neutral or floating determinator is useful, and the tone may be termed the Indeterminator (symbol Ð).

(Tonic-Solfa note: F species, Mae, C species, Doe.)

It can be distinguished in Staff notation by a square note.

In the same way, the bisection of R presents the symmetrical " Contra-triad " T(Z : R/2 : R), where RT=WP.

Symmetrical Triads are Homochromes, and therefore opposed to the convergent series chords.

The symmetrical bisection of the Octave P(Z : W/2 : W) is found in the E.T. diatonic scale at L : TD : WL and TD : L : WTD. The " diminished fifth " and " augmented fourth " are given thereby.

The homochromal characteristics of these intervals stand in highest contrast to the Seriality of B and R, a fact which is of great importance in tonality.

The " diminished triad " TD : TT : L is approximated by the serial P(5 : 6 : 7), P(14 : 17 : 20), etc.

These are discords, although not particularly disagreeable.

The diminished " Contra-triad " presented by L : L◗ : TD is an E.T. equivalent to a diminished Triad.

It shares the same quality.

Motion of either extreme term outwards one E.T.A. converts the chord into a pure triad.

The Serial triads stand as representatives of concordance; the homochromes, in consequence, typify abstract non-concordance.

There is also another aspect. Serial inherence stands for

definity of Species, which is analogous to Direction, F and C being opposed, both totally and in partials.

The symmetrical tones bisecting W, B, R, etc., are neutral as regards species, since they are ambiguous in either extreme.

Hence, if a direction is posited for them, it must be projected in a " dimension " neutral to either con- or anti-verticate, and may therefore be regarded as orthogonal to both, somewhat after the manner in which $\sqrt{-1}$ is considered geometrically as orthogonal to ± 1.

This aspect introduces the idea of manifold direction, ***i.e.*** Phase, into tonality.

In the arrangement of triads,

$$(L : LD : LT) \quad (P : PD : PT) \quad (T : TD : TT)$$

there are three principal triads which may be called Predominant.

But in this group of triads, considered as a row of tones, there are also two intermediate triad-form chords:—

$$(LD : \genfrac{}{}{0pt}{}{LT}{P} : PD) \quad (PD : T : TD)$$

which represent an " inside out " aspect.

These may be distinguished in relation to the nominant, as Recessive Triads (they are really compound chords).

It is to be noted that they are commute in relative species to their origin.

CHAPTER VII

SUBSECTION 1

PHASE

THE term " Phase " has a variety of meanings in scientific and general language.

In the present instance, the term is used, as in mathematics, to express angular value measured from a given base, an aspect which electrical engineering has made familiar to a large number of people.

The measurement of argument or arc in terms of radius (radian units) is explained in elementary treatises on trigonometry, etc. It is convenient and easy to understand.

The phase of an angle may be expressed in several different ways, but in the above method 2π is the value of radians in a complete period, π radians signifying a complete reversal of linear direction.

Rectangular co-ordinates, orthogonal exponentials, vector expressions, etc., provide other systems of notation, some of which will be mentioned in due course.

The justification for the importation of language borrowed originally from space geometry has already been touched upon.

Its consistency, when properly utilised, will be admitted.

There is, of course, no implication of a metaphysical relation between tonality and space; the terminology and symbolism being only borrowed for the purpose of providing a comprehensible and compendious method of investigation into the problems of tonal relationship.

If the reader can mentally visualise these relations without the use of spacial illustration, it is possible to discard such methods; meanwhile, we may proceed on these lines.

The generality of Phase as an ultimate attribute of Mode will possibly be more or less familiar to the reader; but the rationality of the present method depends upon the applicability of such terminology to any experience of a periodic or recurrent nature, which may be termed a Cycle in the abstract.

Whenever a set of relationships exhibit recurrence or quasi-recurrence of properties (such as numerically expressible values plotted as a curve) the cyclic abstract is evident.

The basis of reference, with recurrent characteristics, is the simplest case. It is represented by the simplest plane-closed curve, the circle, from which all methods of representation are derived.

The expression of phase found in variables constrained to linear projection is necessarily restricted to the signs of opposed direction, plus and minus.

In the process of measurement, the minus sign corresponds to the reversal of a positive projection or operator, which may be brought about by swivelling the direction through an angle of π radians (hemicycle). The repetition of this operation (total swivel through 2π radians) restores the original direction of projection.

In this restricted linear view, no cognisance can be taken of intermediate stages or "phases." All such appear as neutral or indifferent, and magnitudes including them are, in the old language of mathematics, resolvable into "real" and "imaginary" parts.

The operation of reversal need not necessarily involve swivelling (it may be performed by sliding through the zero point, and in other ways), for in three-dimensioned space there is an infinity of planes in which a continuous magnitude can change its direction.

The positive and negative signs are mutually periodic in phase, so that the three independent linear conversities connote three similar mutually orthogonal planes of directional variation:—

(up : down) (right : left) (pro : contra)

The arbitrary allocation of positive direction determines the sign of the semi-period as negative, as well as the chirality or "hand" of the two possible directions in which linear swivelling can take place.

Various cyclic or quasi-cyclic characteristics in tonality will suggest themselves. The predominant case is that of the Octave, in which the tonal nominance recurs at every multiple, *i.e.* at every achrome of an axis in a widening interval.

Recognition of the recurrent characteristics of the Octave is evident in the earliest history of tonality; indeed, one can hardly

conceive the formation of a practical system which does not include it.

It is observed in the earliest glimmerings of systematic musical procedure, and survives, so far, the most daring essays of modern experiment. Possibly some of the more extreme pedantic systems of tuning devised by mathematical enthusiasts may have resulted in false octaves, but such systems cannot have had much practical value.

The achromatic property of the octave has already been referred to in discussing the Binary Logarithm system of pitch measurement, and the system of equal temperament in which only the octaves are justly intoned.

The phase-cyclic systems of tonality are infinite in variety, but attention may be restricted to those between the three Orders of components presented by the concordant Triad, and the general intercommensural system derived by the E.T. extension of the principle.

It is found that the error of tuning 7W=12B, 5W=12R is not great; moreover, since it is well distributed over the pitch range it is not particularly evident.

The Pythagorean system of tuning is used by stringed instruments, which are consequently a little out of exact tune.

The C of the Viola and the E of the Violin do not stand in the simple relationship of a power of two to five, but they are fairly well apart, are of different tint, and can be accommodated by experienced players.

The E.T. Blue=1.4983 . . . $W^{\frac{7}{12}}$

The "just" Blue=1.50000

Consequently, the system of tuning attributed to Pythagoras is not of mere academic interest, but in consequence of the great influence exerted by orchestral music (whose mainstay is the body of strings) is a very practical factor in tonality.

The E.T. system permits of a general commensurality, whereby multiples of intervals belonging to different orders can be "equated" and exchanged for each other.

The number of E.T.A. units in the Octave is twelve, being the least common multiple of 2, 3, 4, and 6.

Moreover, twelve is the sum of the harmonic chrome complements $(5+(4+3))$.

These twelve units represent all the concordant elements of tonality fairly well, together with their fluents; but it should be noted that the Impellent 16/15 (the Scalar) and the Suboscillant 25/24 (the chromatic) semitones are represented by the same value in the Antinominant, although differentiated in Staff notation.

An achromatic state of affairs may be attained by adding together either:—

$$12A \doteq W \text{ (a fluent cycle)}$$
$$\text{or } 12B = 7W; \ 12R = 5W$$

(a Pythagorean chrome cycle applicable on the basis of achromatic reduction).

These intervals are qualified to be the " Grades " of a cycle of twelve steps, which may be represented by a polygon of the same number of equal sides, a Dodecagon.

Remembering that W/2 is in excess of R, and in defect of B, by one E.T.A., it is seen that every alternate step in both forms of cycle will achromatically coincide.

$$2B = (12+2)A \qquad 2R = (12-2)A$$

Hence a combined method of phase symbolisation may be developed, which is applicable to both forms.

Six steps in either dextral or laeval direction bring us to πN, which may be termed the " Diametrical " of the nominant.

This relationship corresponds to that between plus and minus.

Three steps in one direction equal nine in the contrary.

Taken in either direction this brings us to the two " Orthogon " (right-angle) values, which are situate at the quarter and three-quarter points on the arc of circle.

These correspond to the mathematical " imaginaries " $\pm\sqrt{-1}$.

The particular symbolical or graphical representation of cyclic relationship is a matter of device according to purpose and convenience. However, no one wishes to be burdened with more symbolisation than absolutely necessary, and hence any suggested scheme must have some definitely apparent claims for consideration, or it will die a natural death.

Fortunately, the dodecanal system lends itself to a most convenient and obvious method of semigraphic expression.

In the universal arrangement of the dials of our clocks and watches, we have a figured cycle of twelve degrees of arbitrary locus and polarity.

By actual or imaginary rotation of the dial with respect to the arbitrary way of looking at it, we can alter the locus of the figures, and by regarding the dial in an actual or imaginary mirror we can reverse the " polarity " or direction of numerical succession.

Hence we may, with some confidence, employ the dodecanal system of notation to express the phase nomination and relationships of the (E.T.) tonal elements.

Draw a dodecagon and number its angles with the clock figures.

On this figure, the alternate coincidence and opposition (by π) of the two forms of cycle appear at the even and odd numbers respectively.

The first- and second-order chromes are designated in terms of the peripheral grades. The zero order of the whole period does not appear, and may be considered as plotted either on the movable radius, or at right angles to the plane of the diagram.

In this latter view, the cycle can be regarded as the end-on view of a helix, whose convolutions measure each recurring octave.

SUBSECTION 2

PHASE RELATIONSHIP

The development of a system exhibiting structural symmetry, such as the dodecagon, is a matter entirely independent of the dative conditions. For instance, it is quite possible to arrive at the system of crystal forms possible in three, or any number, of dimensions, by purely mathematical processes (*vide* Bravais space lattice). However, the actual form of crystal is determined out of the possibilities, by the generative forces and molecular conditions, etc., involved, and once these are ascertained, the particular development and permutation of crystal morphology become known.

Similarly, in Tonality, the problem is not dependent upon the structural symmetry of numerical conditions involved. These follow upon the selection of any definite system; in the case of tonality, the determinative factors are the conditions of the three-fold acoustics.

The whole notion of a phase aspect of tonal relationship would be of mere academic interest were it not for the light that this view throws upon actual experiences.

The particular phase-cycle corresponding most nearly to actual tonal conditions is the Dodecanal.

This exhibits the greatest generality of expression as regards the relations of tonal elements possible with such a limited number of representatives.

The twelve tones of the Dodecanal cycle may be termed Nomials, to distinguish them from the components of Series and Chords.

The twelve relationships, intervals, or steps may be termed Grades.

It is convenient to use the Pythagorean terms for the Dial illustrations, for reasons which will be apparent.

On this basis the relationships are expressed as follows:—

Zero-Order Relations.—Octaves are represented by whole periods on the Dial.

They may be independently exhibited by the Helical development in three dimensions, shown, if desired, in perspective.

By neglecting achromatism for the time, attention may be confined to the plane diagram.

Species is discriminated by the aspect in which the dial is viewed, either from outside, as usual, or from the inside of the "clock," which reverses the chiral arrangement of the whole.

It is convenient to regard the direct aspect (reading dextrally) as representative of the Fundamental Species.

Conversely, the reversed aspect (as seen through a transparent dial) stands for the Coincidental Species.

First-Order Relations.—Owing to its prominent audentity; the fact that it is the first chrome beyond the octave in the Series; and on account of other general advantages; the Pythagorean interval B (or its applement R) may be used as a Grade Unit.

Apart from other conventions, B may be considered to read to the right in Fundamental Species and R to the left.

In the C Species, the hand of the respective direction is reversed.

SECOND-ORDER RELATIONSHIPS do not appear directly, but are represented by the tempered equivalents in Pythagorean tuning, thus appearing as multiples of First-Order grades (3, 4, 8, and 9).

The first term to the right of the Fundamental Prime is the Tensor, and that to the left is the Laxator.

These are reversed in C Species.

The actual divergence from pure Pythagorean intonation is practically negligible, since the frequencies:—

$$W : B :: 1{\cdot}4983 : 1{\cdot}5000$$

The chromes thus differ but slightly: the approximate ratio of the terminators being 74 : 73, a mistuning of the order (6W + V).

The direction of the Cycle is, according to previous convention, as follows:—

Fundamental	Infra	(B)	Dextral
,,	Ultra	(R)	Laeval
Coincidental	Infra	(B)	Laeval
,,	Ultra	(R)	Dextral

The nomials, therefore, reading outwards from the Prime or Centron (which is the Zero value of each), are:—

INFRA. Zero Tensor.		Prime.	Centron.	ULTRA. Zero Laxator.	
Tensor	T			Laxator	L
Bi-tensor	TT			Bi-laxator	LL
Tri-tensor	TTT			Tri-laxator	LLL
Quadri-tensor	TTTT			Quadri-laxator	LLLL
Quin-tensor	TTTTT			Quinta-laxator	LLLLL
Sexa-tensor	TTTTTT			Sexa-laxator	LLLLLL

this final value in both "polarities" being identical with the Diametrical Prime or "Diacentron."

Roman numerals may be used to denote these values.

The dual concomitance of Species may be shown on the same Pythagorean cycle by the following values.

The corresponding members, on the "natural" key basis, may be given in Tonic-Solfa notation, in both the Nominant and Concomitant Commute (F and C) species:—

	Species.			Species.	
Member.	F.	C.	Member.	F.	C.
T_o	Doh	Me	L_o	Doh	Me
T_I	Soh	Lah	L_I	Fah	Te
T_{II}	Ray	Ray	L_{II}	Ta	Fe
T_{III}	Lah	Soh	L_{III}	Ma	De
T_{IV}	Me	Doh	L_{IV}	La	Se
T_V	Te	Fah	L_V	Ra	Re
T_{VI}	Fe	Ta	L_{VI}	Sa	Le

In the above table the " coinciding " or " conjugate " nomials are situate at the Bi-tensor and Quadri-laxator.

The reason for this arbitrary relationship is obvious.

The approximation to second-order chromes at the Tri- and Quadri-nomials is also to be noted.

C. *Second Order.*—Following preceding methods, the Prime may be regarded as a Zero determinator, D_0.

" Direction " is determined by the approximate commatic equation 4B=G; the Quadri-tensor being regarded as representing the co-specified or Educt determinator.

There are only three values in any one cycle, forming the limiting or trigonal expression; there being four independent arrangements in the dodecagon. The Centron is the prime or zero determinator.

The determinator is attained by the smallest pythagorean journey in the same direction as the first-order chrome is plotted, *i.e.* that of the original series: being dextral for fundamental and laeval for coincidental species.

The permuted determinator is attained by two pythagorean steps in the contrary direction; it is therefore " adduct " in species, relative to the matrix.

In the E.T. equation it is seen that the bi-determinators of either chirality are the respective determinators of the opposite species, according to direction, thus:—

Se=La, and Fa=Me.

The infra-second-order cycle is thus completed by the three nomials, which, in Tonic-Solfa terms, appear as:—

Fundamental Species

Adduct.	Zero.	Educt.
Fa . . . La	Doh	Me . . . Se

Coincidental Species

Educt.	Zero.	Adduct.
Fa . . . Doh	Me	Se . . . Tee (Doh)

The corresponding values in each species are noted.

The Second-order Ultra chrome V forms a tetragonal cycle, of which there are three arrangements in the dodecagon.

The four nomials are, with respect to the centron, in F Species:—

Prime or Zero

Dextral orthogon	Tri-tensor
Diametrical Prime	Sexa-tensor
Laeval orthogon	Nona-tensor

and conversely in C Species.

The Tetragonal Cycle has as unit of grade the "inferior" limit of chromality (which is practically the superior limit of fluence) in which the same nomials appear, but with reversed orthogons, owing to the equational form in which:—

$$3B=Y \qquad 3A=V$$

The ratios of the second to first order chromes may be put in the form of two Subcycles:—

$$V=3A \text{ or } Y=9A$$
$$G=4A \qquad M=4O$$

The grades in this case are fluent, but can be pictured in a manner similar to the general cycle, in which, of course, they inhere.

Fluent Relationships.—(A) *Zero Order.*—The achromatic interval of the octave is represented by one complete period of the cycle, as in the Pythagorean system.

In accordance with a previous convention, Fluents are nominated from the Chromes of which they are the First differences. Thus their nominal order and their species follow the indication generally adopted for chromal phase values.

This value has the disadvantage of being manifold (Oscillants equal $Y-B$, $B-R$, $R-V$). It is therefore often more convenient to regard fluents altogether independently of any chromal origin, in which case the pitch range affords the means of discriminating

two directions, viz. rising (aclinear) and falling (declinear), the direction being reversed in the case of ultra-fluents, viz. $W-K_2$.

The rationality of this aspect is seen in Successive Tonality, where the " flexion " is considered.

In the E.T. system, there are only two classes of fluents to consider, viz. Oscillants, 6 to the octave, and Antinominants, 12 to the octave.

The aspect in which the Oscillant is regarded as derived from the First-Order chromes is useful as a means of nominating order, etc. It is based upon the first appearance of the interval in the Series, viz. P(8 : 9).

(B) *Oscillants.*—The oscillant is the unit of a hexagonal cycle of phase, whose principal characteristic is an " oscillation " of the successive multiples between concordant and discordant intervals.

Since $2B=W+O$, and $2R=W-O$, the Hexagonal cycle corresponds to an alternate Pythagorean, and the members are achromatically identical at the alternate steps from the Prime of:—

Dextral.	Laeval.
Bi-tensor	Bi-laxator
Quadri-tensor	Quadri-laxator
Sexa-tensor	Sexa-laxator

The oscillant may also be regarded as the hemichrome G/2.

Hence, in the subcycle of G it appears as the Diametrical Interval of projection and retraction.

The diametrical of the Bitensor (with respect to W) is D_{II}, which bisects the ultra-second-order chrome M, and is therefore the " diametrical " of the complementary subcycle of M, which comprises four oscillants.

$$M/2=G.$$

Hence the ratio between the Intervals is $M : G=2$.

(C).—The Antinominant is the fluent unit of grade in the Dodecanal Cycle, which thus corresponds to the Pythagorean chrome.

It forms the most general E.T. Cycle of 12 steps, each of which is alternately identical with, or diametrical to the corresponding (achromatic) Pythagorean members.

Hence every member of a general dodecanal cycle represents

a dual validity, and the statement of alternate coincidences and differences appears as a pair of interpolated hexagonal cycles of identities and hemichromes respectively.

It is convenient to regard only the aclinear and declinear distinction of the antinominant cycle, reserving the idea of polarity to the Pythagorean form.

It is to be noticed that starting from a given Prime, the Fundamental alternate coincidences read:—Dextral-Aclinear and Laeval-Declinear, respectively; the converse being the case with the Coincidental alternates.

The direction of the coincidences is reversed upon substitution of the ultra antinominant (W–A) for the infra, or simple fluent.

(The Super-fluent corresponds with the Chromal V and its cycle is consequently identical with the tetragonal form of that chrome.)

The Fluents are members of Subcycles as follows:—

Complete Period.	Elements.
G	Tetragonal Antinominants
M	Tetragonal Oscillants
V	Trigonal Antinominants
Y	Trigonal Super-fluents

The generality of the G period cycle, and the fact of its approximate identity to the Externominantal (Parasyntonic) Tract, render this cycle particularly prominent.

Tetragonal forms may be considered in the same way as Pythagorean criticals.

They comprise the dextral and laeval orthogonals together with a centron and its diametrical.

Trigonal forms are conveniently regarded from the aspect of the similar chromal form, viz.:—

$$W=3G \qquad 2W=3M$$

This form, as the limiting "bi-manifold" (two straight lines cannot enclose a space), is particularly interesting.

It is seen that, in proceeding round the cycle, an enharmonic change of chirality is effected. This gives a peculiar effect in actual practice, which may be looked upon as one of the tonal acquisitions we owe to the E.T.

This entire reversal of chirality is one of the most extreme

(from a graded point of view) cases of enharmonic modulation in actual use.

Its logical extension, applied to the " composite symmetrical " type of septomial scale (aclinear or up-melodic minor), has provided the celebrated hexatonic scale of oscillants, with which some modern composers have obtained good effects.

The peculiar change of chirality experienced in this scale (apart from its particular style of harmonisation) is akin to that found in traversing the Trigonal Subcycle. It is seen to really consist in the co-progression of two independent cycles; one, the ordinary cycle of pythagorean or antinominant chirality, and the other, a cycle of phase itself, apparently orthogonal to the original cycle, and thus giving the effect of another " dimension."

This matter will be discussed in the next Section.

The Hypo-chromes 7/6 form an approximately pentagonal cycle within the octave, and the Super-fluent 8/7 a heptagonal value; but, since these values are incommensurable to the E.T. system (being capable of approximation by summing alternate values in excess and defect), they are of mere academical interest.

SUBSECTION 3

PHASE EXPRESSION

The class of tonal relation, coming under consideration in the aspect of Phase, differs in character from the serial system already considered, in being non-convergent, *i.e.* Homochromal.

The idea of a serial system essentially involves the convergence and divergence of included elements: the recurrence of order elements being necessarily marked by similar convergence and divergence of specific attributes.

Upon proceeding outwards from a Prime, as soon as the convergent or divergent differentia become indiscriminate to the human perception, the series-form is no longer acceptable as a true state of affairs, although its theoretical significance extends *ad infinitum*.

Physiologically and psychologically, series " vergence " becomes nebulous and invalid at a certain region.

The contrasting attributes of Con- and Di-vergence are ab-

stracted under the name of Species, with its "partial" aspects in each particular order, comprising the Vertical, Chiral or Polar, and Permutable aspects.

Definity of species is the most persistent characteristic of tonality, one which appears to survive even the most extreme treatment of musical sound.

This does not necessarily imply limitation of specific definity to any conventional or notational portions of a musical work, but that the two species of Triad remain the anchors of concordance.

The "coherent" relationships of serial tonality are based upon simultaneous phenomena.

The Harmonic series, and the regional experiential discriminants of chrome, fluent, and limen, are essentially experiences of simultaneous tonality, which remain associated with the chordance aspect of tonal relations.

In Phase relations, the same material is encountered but in a different aspect, not less fundamental than the serial, which may perhaps appear as experientially predominant.

The characteristic of Phase relationship is the non-convergence or homochromality of membership, with the consequent independence from species which obtains in the abstract form, although the experiences themselves are inseparable.

Homochromality is also distinguished from serial con- and di-vergence in being independent of simultaneous conditions.

Its materia may indeed be derived from the serial forms, but the equal steps or divisions of the pitch range (or logarithmic projection of frequencies) are due to the particular conditions under which the "Fechner and Weber" principle becomes effective, viz. the transposable persistence of chrome identity and the possibility of chrome step recurrence.

Serial relations are those to which our ears passively submit. Phase relations are the outcome of an active operation, viz. the piling up, or stepping over, a number of identical intervals, which may be either arbitrary, prescribed, or approximative.

Purely arbitrary selection of Phase units does not appear evident in the music of Western civilisation, but traces of it may be noted in the Chinese and Persian methods derived from a somewhat sophisticated experience.

The grades of the cycle are derived from the Series, and are made commensural by the E.T.

The grade may, however, be attained by the arbitrary division of an octave (or any other assigned interval) into a number of equal intervals. When this number equals twelve, the approximation to the early serial values is very close.

Like all the manifestations of human existence, Tonality has had many ancestors; the process of grafting formality upon a primary stock of emotional expression, has resulted in many peculiar characteristics which persist in its present form.

The harmonic elements of homochromality have their origin in concordance. Equifluental elements are more concerned with the presentation of the "shape" of a melody, an entity which may be transposed unchanged over a considerable range of pitch.

The chromal elements of the concordant series are distinguished by their order; hence the Harmonic Homochromes may be divided into three classes, according to the order-chrome to which their units approximate.

The Cyclical or Recurrent element, upon which Phase depends, rests upon the approximate coincidence between integral multiples of independent units.

These permit the formulation of a number of approximate equations, in which the conditions are the preponderant achromatic recurrency of the octave and the commensural possibility of the Zero, First, and Second orders of chromal units.

The illustration, by means of spacial terms, has been developed into a method of nomenclature that appears particularly suitable for the purpose of enabling the generality of relationship to be comprehended.

The basis upon which we regard tones, not as Euclidian points but as centres of limited regions on the pitch range, enables the approximation of cyclicality to be of real value in expressing tonal experiences.

Out of the elements of quasi-equation we derive the most general and economical system of twelve steps to the cycle, which appears in the dual form of the "Pythagorean" (Chromal) and "Antinominant" (Fluent) period of the octave.

The alternate coincidence, and difference by half a period (W/2) of the nomials belonging to the respective general systems, enables one general schema to be adopted, and the many allied characteristics of the dual systems to be compared.

In adopting the plane circle as the basis of notation, with the dial figuration as a particular method of representation, we are able to utilise several well-known processes of divisional expression.

The extent of phase is measured by the Argument, or Arc, of the cycle.

Since the units are fixed values, the circular dial is replaced by a polygon of twelve grades, each of which has a certain magnitude and a definite direction.

A grade is thus a "Vector" expression; and can be denoted in any of the several ways by which such a composite value is conventionally expressed.

To examine these conditions, we may conveniently plot out the dodecanal "graph" representation of the twelve nomials; utilising the arbitrary arrangement of the dial familiar from our clocks, as well as the convention of ascribing the direct aspect to the predominant vertical experience, viz. Fundamentality, in the Harmonic serial form.

The notation employed is conveniently that of the Pythagorean or Chromal system; which lends itself to the direct examination of chords.

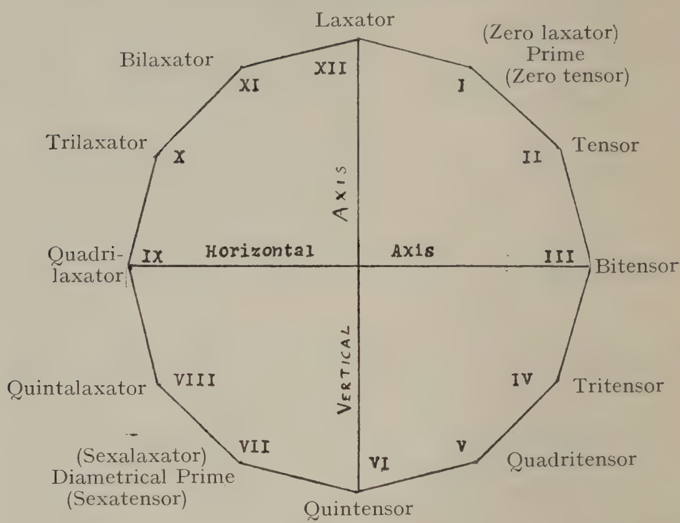
Every nomial can be expressed in Antinominantal form, by merely prefixing the diametrical symbol (π) to the odd or even alternate steps.

$-A_{VI}$	$L_{VI}\pi$	$-N$
$-A_{V}$	πL_{V}	$-A$
$-A_{IV}$	$L_{IV}\pi$	$-A_{II}$
$-A_{III}$	πL_{III}	$-A_{III}$
$-A_{II}$	$L_{II}\pi$	$-A_{IV}$
$-A$	πL	$-A_{V}$
N	$P\pi$	$\mp A_{VI}$
$+A$	πT	$+A_{V}$
$+A_{II}$	$T_{II}\pi$	$+A_{IV}$
$+A_{III}$	πT_{III}	$+A_{III}$
$+A_{IV}$	$T_{IV}\pi$	$-A_{II}$
$+A_{V}$	πT_{V}	$-A$
$+A_{VI}$	$T_{VI}\pi$	$-N$

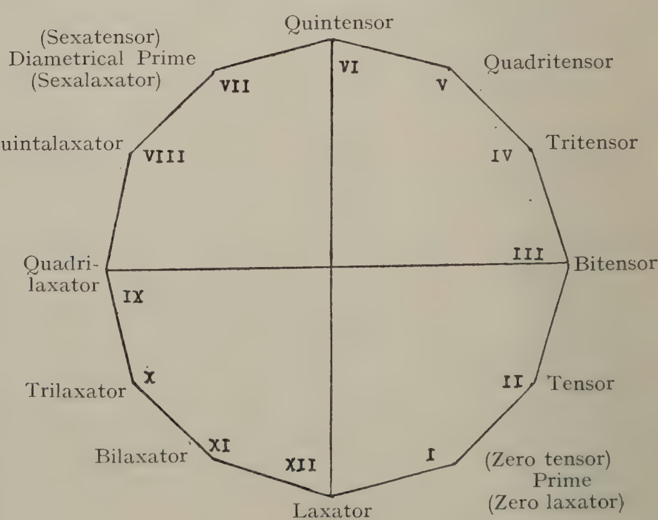
B units of grade.

It should be noted that $\pm A_0 = N$.

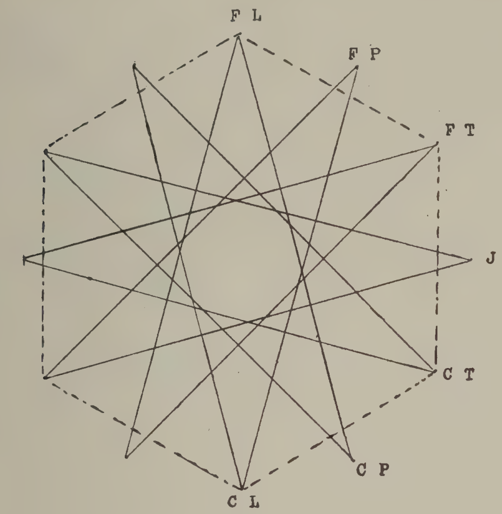
FUNDAMENTAL ASPECT

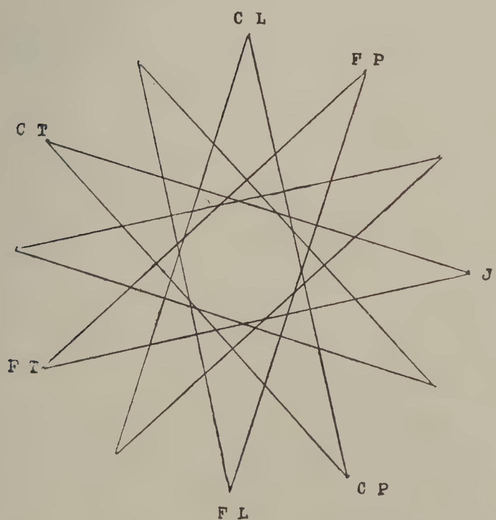


COINCIDENTAL ASPECT



The representation of one type in terms of the other appears:—





The preceding diagrams show the Fundamental Graph and the "Invert" image obtained by turning the plane over about the "horizontal" axis to form the coincidental figure.

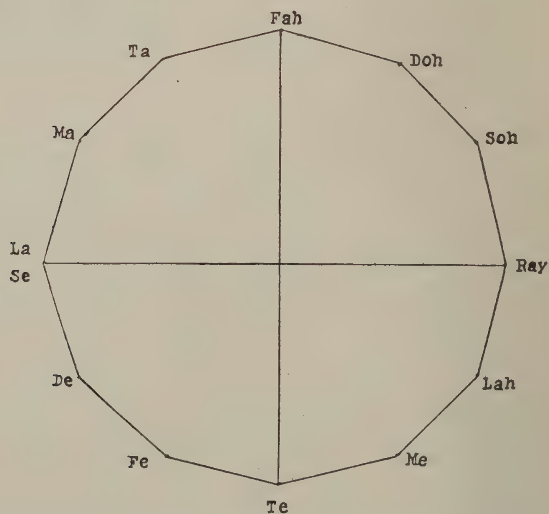
It will be noted that the Dextral and Laeval Hemicycles are not displaced, but the Superior and Inferior Hemicycles are transposed about a fixed axis, whose terminators, the Bitensor and its "Diametrical," remain common to both species.

These values are therefore the vanishing points of specific discrimination, contrasting with the Laxatorial Axis of maximum species.

The corresponding change about the Laxatorial Axis gives rise to another class of relationships shortly to be discussed.

We may combine the two graphs by using the Tonic-Solfa names of the Pythagorean nomials.

To convert any set of alternate values into Antinominants, the tones should be read as their diametricals, *i.e.* transposed a "Tritone."



At once discrimination becomes possible about the Vertical Axis.

The seven nomials composing the Dextral Hemicycle, together with the two "horns" Fah and Te, are the members of the diatonic, or natural scale.

The five complementary nomials on the laeval side are

the "excluded" or "accidental" elements of the range of modulation.

The Vertical Axis Fah–Te is, in E.T., a hemichrome W/2, about which a diametrical phase transformation can occur.

This is the maximum displacement of a "Key" in E.T. Tonality, and is experientially the most "neutral" located progression that a simple chord or passage can effect.

Referring to Triadal and Key progression only, irrespective of compound chords to be considered later, it may be noted that the Phonic Continuity of a progression or relationship is at its minimum in the case of this perversion about the vertical axis.

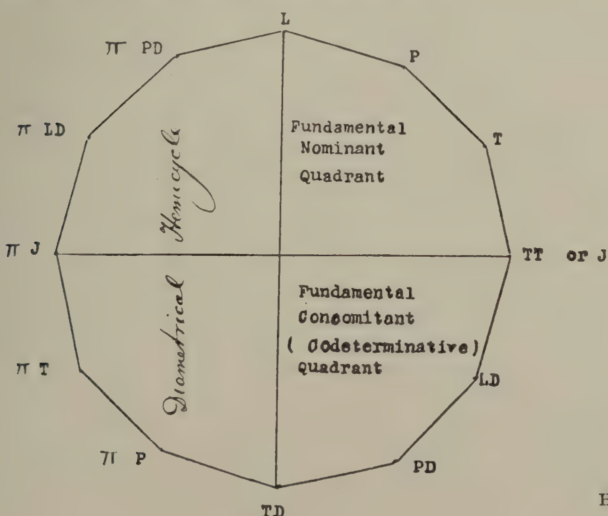
The axis therefore represents the vanishing points of Phonic Continuity, and its terminatives constitute the outposts of Diatonic Location.

The Vertical and Horizontal Hemicycles are seen to be associated with experiences which can be recognised generally, and the respective orthogonal axes correspond to the constant elements in definite Tonal contrasts.

Hence the Cyclic Graph is no mere device, but a more or less definite representation of tonal experiences possessing considerable evidence.

The quadrantal aspect of the general cycle may now be considered.

This may be put into two forms, as follows:—

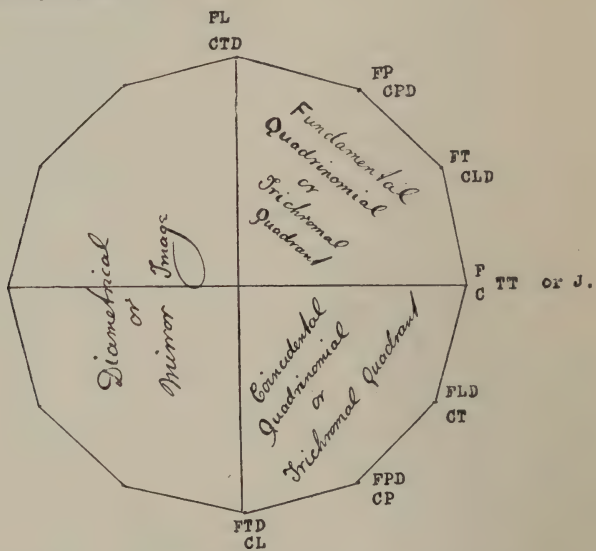


H

In the above diagram, it is to be noted that the Superior-dextral quadrant contains the Zero and First-order elements, while the Lower dextral contains their concomitant determinators.

This is the arbitrary representation of the Fundamental Triadal grouping; the corresponding Coincidental arrangement having the Second-order components in the upper dextral, and the prime, laxator, and tensor in the lower dextral quadrants respectively.

Consequently, the mutual arrangement may be expressed in the following figure:—



This particular arrangement, corresponding to the "Commatic" equation:—

$$\left.\begin{matrix} F \\ C \end{matrix}\right\} = \left\{\begin{matrix} C_5 \\ F_5 \end{matrix}\right.$$

may be known as the "Concomitant" relationship.

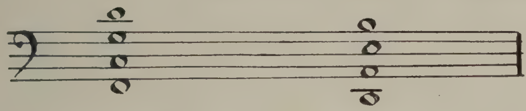
Changes of Species in this type are therefore known as Concomitant Commutation.

In the Concomitant system $F_9 = C_9$ (also $\pi F_9 = \pi C_9$).

The mutual bitensor acts as the centroid of the equated systems, and may therefore be known as the Yoke (symbol J, Jugator).

It may be noted that the Yoke is the only member of the Just Hemicycle which has a dual enharmonic pitch value.

Concomitant relationship is expressible in Staff notation, where the two groups of simultaneities are each of first order among themselves, but are mutually of second order in relationship.



F Trichromal. C Trichromal.

Upon tuning the triads justly to 1 : 3 : 5, it is noted that the upper terminal of the F group differs from the lower terminal of the C group, although notationally identical (achromatically). The difference is the Comma, 81 : 80, whose "grade of mistuning" is 6W+G.

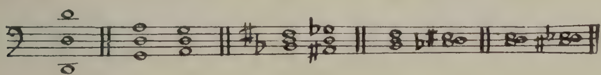
This interval approximates to one-fifth of an E.T.A., and, though distinctly appreciable, is not sufficiently "out of tune" to have made the great works of music impossible.

It may thus be definitely included in the Liminal class of interval.

The tone J is thus the "mean-tone" of the two concomitant species, and is the centre of the dual hemicycle.

It divides the diatonic scale into symmetrical chromal divisions, notationally and on the keyboard, and, as pointed out by Oettingen, it may be compared to a Mirror in which the concomitant commute image of any chrome is reflected.

This may be illustrated in Staff notation, as under:—



Any such group of tones is the minimum individual Homochrome composed of three nomials.

It may be known as a "Trinomial," and the two trinomials of the typical chromal and fluent types are the especial prominent group relations:—

$$\begin{array}{rcl} L : & P : & T \\ \pi L : & P : & \pi T \\ L : & \pi P : & T \end{array}$$

The Trinomial consists of a Centron, and its two "Poles" respectively dextral and laeval.

In the case of second-order chromes, the "polar" elements form chromes which conflict with the origin.

Hence the term "Trinomial" may be particularly restricted to the cases of First-order and Antinominant relations, whose poles are "oscillant" with respect to each other in both cases.

The Chrome Units, (P : T) and (P : L), are seen to be the polar projectors. With respect to T, P is Laeval, while with respect to L, P is Dextral in polarity.

Consequently, the Trinomial represents an equation of Polarity about the Centron P, and this, in both Pythagorean and Antinominant types, is the simplest "Matrical system" of Tonality.

The Concomitant relationship of the "Poles" of the Yoke may be expressed in the following sections of the Pythagorean Cycle, in which the upper members are Fundamental, and the lower Coincidental:—

Ter-nomial System

$$\left.\frac{\text{Soh}}{\text{Me}}\right\}\quad \text{Ray}\quad \left\{\frac{\text{Lah}}{\text{Soh}}\right.$$

Quinta-nomial System

$$\frac{\text{Doh}}{\text{Lah}}\quad \left.\frac{\text{Soh}}{\text{Me}}\right\}\quad \text{Ray}\quad \left\{\frac{\text{Lah}}{\text{Soh}}\right.\quad \frac{\text{Me}}{\text{Doh}}$$

Septa-nomial System (Pythagorean diatonic)

$$\frac{\text{Fah}}{\text{Te}}\quad \frac{\text{Doh}}{\text{Lah}}\quad \left.\frac{\text{Soh}}{\text{Me}}\right\}\quad \text{Ray}\quad \left\{\frac{\text{Lah}}{\text{Soh}}\right.\quad \frac{\text{Me}}{\text{Doh}}\quad \frac{\text{Te}}{\text{Fah}}$$

It has been held by some musical historians that the Diatonic scale was evolved upon these lines; but the evidence on this point is conflicting: the conditions were no doubt contributory to any process of parallel type.

The terminators of the Heptonal System, or Hemicycle, are diametrical with respect to each other, formulating a maximum contrast to the Serial and Coherent relationships of the Concordant triad, as well as to the "graded" nomiality of the Cycle.

Practically, the criterion of Species, and of Ortho-species (Diametrical discrimination), turns upon the acceptance of the same interval, W/2, as either a "diminished B" or an "augmented R."

Under conditions which do not permit definite allocation either one way or the other, the discriminance of B and R is destroyed, and therefore the Series becomes anomalous.

Hence the contrasting character of the E.T. Tritone.

A particular case, in which this aspect becomes of experiential importance, is the approximation to E.T. tuning presented by the odd series up to the 13th member on the Tensor (and, to a lesser extent, upon the Laxator).

In the case of the Fundamental Tensor, it is possible to hear this series in the " Klang," forming a chord which approximates to:—

Soh : Te : Ray : Fah : Lah : (Doh) : (Me)

Assuming Ray as unit frequency, the above tones, in just intonation, appear in the Concomitant Species as follows, taking Soh as Fp, and Lah as Cp:—

	Soh	:	Te	:	Ray	:	Fah	:	Lah
$F\Sigma_1^9$	1		5		3		7		9
Frequencies .	$^2/_3$	:	$^5/_6$	:	1	:	$^7/_6$	:	$^3/_4$
$C\Sigma_9^1$	9		7		3		5		1
Frequencies .	$^4/_3$	:	$^6/_7$	:	1	:	$^6/_5$	:	$^3/_2$

In equal temperament these differences of characteristic species vanish, and the " mistuning " observed appears as a purely physical phenomenon, tending towards an equalisation of Series.

Hence Ray, as the centre of the " homochromic tritone," appears as the conjunctive tone at which the Series verges towards the concomitant Primes of either Species.

Soh : Te : Ray is accepted as an unmistakable fundamental triad, while Ray : Fah : Lah is just as unmistakably a coincidental triad; and it is to the Vertication characteristics that we owe the bias of Fundamentality observed in the Pentad group in question.

This " Bias " is still further pronounced when the eleventh series member of the Fundamental " klang " is added: under which circumstances the Hexad suggests:—

Soh : Te : Ray : : Fah : Lah : Doh

i.e. the Tensor and Laxator (Polar) Triads.

In consequence of its peculiar conjunctive position, the com-

matic tone Ray (the mean between extreme species) acquires a Neutrality in respect of Species; and, as we have already observed, in systems where no obvious criteria appear to discriminate between its species-inherence, it is the tone that becomes naturally the Mean or Middle of Tonal symmetry.

To adopt a somewhat poetic simile: the modern Harmonic (*i.e.* Tertriadal) system of chordance is based upon a sworn allegiance to a king (the Prime) and his ministers on right and left of the throne (the dextral and laeval poles). The kingdom is constituted by synergetic blending of elements, due to a policy of actively employing coherent characteristics.

The older "Modal system" (as far as homogeneous practice obtains) was based upon the Homochromal characteristics of the "Pythagorean Hemicycle," itself a condensed generalisation of the Melodic Shape-basis, and may be compared with a republican constitution; of which Ray is the elected president, who balances the power of the symmetrically opposed parties of the state (the Species).

The aspect of Tonality which may be known as Ortho-Species is the discrimination between the two Hemicycles, Dextral and Laeval.

As a matter of fact, we note that the Diatonic elements of a "Key" are contained by the Dextral Hemicycle plus the two "horns" of the crescent; *i.e.* the "Natural" components (white digitals of the basic keyboard system).

The Translaxatorial, or Diametrical, components include the five laeval "accidentals" (black digitals):—

This view of ortho-species is, of course, merely an abstraction holding in the Phase system. It must not be forgotten that however an abstract view we may take of the "Conversities of Symmetry," it is impossible to eliminate from actual experience the superior audentity of the Fundamental species, the infra-first-order, and the infra-infra-second-order, chromes.

The commensural aspect is one which is particularly evident in the Phase relationships of Tonal and Chromal elements.

The successive tone relationships in the Antinominant Dodecanal appear as follows:—

$$Z : A_i : O_i \mid V : G : R \mid \pi \mid M : Y \mid O_u : A_u \mid W$$

(Z stands for Zero)

presenting two groups of Fluents, separated by a central chromal core about the hemicyclic π.

The question of a commensural system other than the dodecanal has received much attention at the hands of musicians, philosophers, and scientists; from various æsthetic, practical, and logical standpoints.

An enormous literature has accumulated around the subject, which may be consulted with advantage.

By Janko's method (see Stumpf's *Akustuk u. Musikwissenschaft*, vol. iii. 1901) of reducing the frequency ratios to Log. base 2, and multiplying these by successive integral numbers until the value approaches a whole number (Achrome), we find the triadal values for the dodecanal, and minimum cycles giving nearer approximations to just tuning, as follows; the numbers being the E.T. units in each chrome:—

	W	B	G
Dodecanal Cycle	12	7	4
Janko's Cycle	41	24	13
Mercator-Bosanquet Cycle .	53	31	17

The unwieldiness and practical intractability of the 41 and 53, *et ultra*, cycles, leaves the dodecanal in sole possession of the field as the most general economical system.

The gap between 12 and 41 no doubt contributes to this predominance, such values as may be found between being decidedly unsuitable.

The Huyghens Cycle of 19 gives better values for second-order chromes, but is considerably out of tune with those of first-order.

The E.T. Dodecanal division of the Octave is seen to be not only an economic compromise, designed to obtain the greatest intercommensural generality with the minimum number of elements, but also a guardian of the sharp distinction between the various Orders of Chromality and Fluence.

If the Pseudo and Interchromal values, such as the natural seventh, were introduced into any system, the limitations of the dodecanal system would be indeed transcended, and new material placed at the disposal of the composer, but the system of Tonal determinacy would lose some of its practical definition. This is, perhaps, not out of sympathy with the apparent tendency

of modern tonal development; but, at the same time, we have to admit that the iron-bound limitations of the older tonal system have provided a fruitful path in the past.

The application of mechanism to keyboard instruments, with a view to securing a nearer approximation to just intonation, has engaged the attention and valuable services of several inventors.

So far, the resulting improvements have not met with much recognition, but there is always the possibility of simplification, and in these days we may look forward to the application of electrical and other devices to secure the grand effects of pure harmonies without involving either excessive mechanical complexity or special technical ability on the part of the performer (or controller) of such instruments.

CHAPTER VIII

SUBSECTION 1

POLARITY

THE Pythagorean system has the particular advantage of dividing up into two hemicycles of six grades and four quadrants of three grades, independently of each other, each of which constitutes a group of allied characteristics.

It is found convenient to regard the whole cycle as capable of analysis into, or synthesis from, the four quadrants, viz.:—

(1) The Nominantal, containing the Prime.

(2) The Coincidental or Co-determinative.

(3) and (4) Their respective "diametricals."

Each quadrant is thus a Trichromal, consisting of a central unit, with its positive and negative companions (poles).

The relative position of the polar units is determined by the direction of the central interval, *i.e.* according to the disposal of the tone terminators of this chrome.

In such a case, once the direction of the centron unit of the nominantal quadrant is known, the rest of the cycle follows the species; and, by means of the operator π, can be transformed into either pair (odd or even) of Pythagorean or Antinominantal forms.

Indication of Phase on this system would appear as simple as it is possible to express dodecanal relations.

The cycle may be illustrated by a ring of twelve bar magnets, placed with opposite poles in contact; the field being thus equated around the dodecagon.

Each element in this analogy is "directed" as regards magnetic polarity.

The entire ring can be reversed in a plane by either:—

(1) Turning the whole over.

(2) Separately reversing each element.

Each unit appears as a directed grade equated by its eleven companions. This is equivalent to saying that one "sharp" equals eleven "flats" and *vice versa.*

The analogy may be extended to the three-dimensioned field formed in space about a bar magnet. This is seen to be represented by the "integral" of closed curves, whose mesial plane section (revealed by iron filings on paper) consists of figure-of-eight shaped loops forming conjugate cycles of opposed chirality.

The limit of loop on both sides is the infinite extension of lines of force in the prolongation of the axis, and the zero limit is seen by the convergence of the loops to the magnet itself.

Commutation may, therefore, be attained by either rotating a loop through half a circle out of the plane of the paper, or by: (1) Shrinkage to zero followed by expansion to the opposite side; (2) expansion to infinity followed by contraction to the opposite or conjugate image.

These somewhat fantastic illustrations serve to show how independent coexistent values, related in phase, can be continuously transformed in phase by commutation.

SUBSECTION 2

RESOLUTION AND TRANSFORMATION OF PHASE

Orthogonal Resolution of Phase.—It is, of course, a matter of convenience as to which of the two general methods of phase expression is employed.

The angular, or argumental, expression in terms of "Radians" is convenient in many ways, but we may now consider the resolution of the Dodecagon into orthogonal components, these being the projected values of any term upon the two principal axes of the cycle.

The method of signs adopted in Trigonometry is convenient. In this, the arbitrary rule is that Dextral and Superior values are reckoned positive, while Laeval and Inferior values become negative.

The ratio of the horizontal and vertical components is called the trigonometrical Tangent (Tan.), and expressed as a fraction whose numerator is the vertical (Projection on the Laxatorial Axis), and whose denominator the horizontal (Projection on the Yoke axis) component of Phase resolution.

The four quadrants thus become differentiated by the signs of the pairs of resolutes, as follows:—

		Symbol.
Fundamental Nominantal (Superior Dextral)	$+$ $+$	
Fundamental Concomitant (Inferior Dextral)	$-$ $+$	
Diametrical (Superior)	$+$ $-$	
Diametrical (Inferior)	$-$ $-$	

The respective ratios, including the unit Radius (Achromatic component), may be expressed generally in trigonometrical terms.

The rationale of the application of this method of expression to Tonal phase is of real value in representing the change of "order" effected by the tempered equation of units.

We note:—

$$3B = W + Y \qquad 3R = W + V$$
$$3A = V \qquad 3A = 2W + V$$

i.e. that the quarter turn is equivalent to an "ultra" second-order chrome.

The process may be seen in an operational aspect by considering the result of successively acting upon the Laxator with step operators:—

$$L \times \frac{(LP)}{L} = LT$$

$$L \times \frac{(LP)}{L} \times \frac{(PT)}{P} = LTT$$

$$L \times \frac{(LP)}{L} \times \frac{(PT)}{P} \times \frac{(TJ)}{T} = LD$$

and this process may be considered as repeated by prefixing an orthogonal operator; presenting the following values:—

Tonic-Solfa Notation.		Quadrants. ‡	∓	±	⌶
Doh		P	:	:	:
De	Ra	:	:	:	T
Ray		J	J	:	:
Re	Ma	:	:	T	:
Me		:	P	:	:
Fah		L	:	:	:
Fe		:	:	:	P
Soh		T	:	:	:
Se	La	:	:	L	L
Lah		:	T	:	:
Le	Ta	:	:	P	:
Te		:	L	:	:

In the expression of the whole cycle as the sum of four quadrants, a condensation of representation is noted.

$$\left.\begin{array}{lllll} & & \text{Quadrant.} & & \\ \text{Dextral} & \text{Super} & \ddagger & \text{comprises} & F \\ \text{,,} & \text{Sub} & \mp & \text{,,} & C \\ \text{Laeval} & \text{Sub} & \text{⌶} & \text{,,} & \pi F \\ \text{,,} & \text{Super} & \pm & \text{,,} & \pi C \end{array}\right\} (L : P : T : J)$$

The four quadrants may also be expressed with respect to a nominating series, in the "orders" of elements.

Quadrant.	F Species.	C Species.	(Fundamental) Notes in Natural Key.			
‡	Zero-first	Second	Fah	Doh	Soh	Ray
∓	Second	Zero-first	Ray	Lah	Me	Te
⌶	Zero-first diam.	Permuted Second	Te	Fe	De	Se
±	Permuted Second	Zero-first diam.	La	Ma	Ta	Fah

The terminals of a quadrant mark the points where Tensorial and Laxatorial Polarity merges into Determinal Chromality.

In the Antinominant form, the same points represent the transforming criticals between Fluence and Second-order chromality.

Hence the Orthogonal expression of Phase corresponds to a real experiential aspect, and is not merely a mathematical elaboration.

In the orthogonal aspect, the radius of the cycle is the constant element persisting in a Tonal system, being the conserved element which is unaltered by translation of the whole cycle over the pitch range of variability. Consequently, the Trigonometrical

expressions, Sine and Cosine, may be used to express the projections on the Laxatorial and Yoke diameters respectively.

The Radius also corresponds to the achromatic freedom which holds throughout the abstract relationships expressed.

The vertical ordinate corresponds to the $\left.\begin{matrix}F\\C\end{matrix}\right\}$ Laxator axis of the cycle.

This may be regarded as a line joining the opposite extremes of the two Species, viz. FL as maximum fundamental and minimum coincidental; and *vice versa* with CL.

The line joining the Prime to its educt determinator

$$\left\{\begin{matrix}\text{FP to FD}\\\text{CD to CP}\end{matrix}\right\}$$

is parallel to this axis.

The horizontal abscissa corresponds to the Yoke axis $\left.\begin{matrix}Z\\\pi\end{matrix}\right\}$ J.

The characteristics of the Dextral hemicycle, and the contrasting Laeval complement, are similarly presented in maximum and minimum by the respective Yokes.

The tangent expressions thus correspond to the projections on the L/J ratio of axes; while, in accordance with this view, "Sine" and "Cosine" expressions would involve the Zero element together with one axis, viz.:—

Sine=L/Z Cosine=J/Z

The arithmetical values of the arguments $\pi/6$, $\pi/3$, and $\pi/2$, are respectively (for Cosine, Sine reversed) .866, .5, .0, giving an "umbral measurement" of Audentity as follows.

In Pythagorean terms:—

	Order Audentity. First.	Second.
P : T	1.0	0
P : T^{II}	.866	.5
P : T^{III}	.5	.866
P : T^{IV}	.0	1.0

In Fluent form we get:—

	Fluence (adherence).	Chordance (coherence).
P+A	1.0	0
P+A^{II} (P+O)	.866	.5
P+A^{III} (P+V)	.5	.866
P+A^{IV} (P+G)	.0	1.0

And, in just intonation $(P : T^{IV}) : (P + A^{IV}) :: 81 : 80$.

Each element of a triad, on the above aspect, appears as an orthogonal component of a triplicate manifold: or, as we should say in the language of quaternions: $iP + jT + kD$; where i represents the vertical, j the polar, and k the permutable, elements of Partial Species discrimination.

The two types of transformation (odd and even) possible between the Pythagorean and Antinominant Systems, and *vice versa,* give rise to a pair of Hexagons of coinciding members, about which the intermediate values undulate with a phase difference of π.

This might be represented by a "wave" diagram, in which the nodes stand for coinciding, and the loops for differing, members.

The two types may be denoted respectively by the coincidence falling upon either the Laxatorial or Yoke members.

For the seven dextral members, we thus obtain the two following groupings:—

<table>
<tr><td>Pythagorean .</td><td>FL</td><td rowspan="2">FP</td><td>FT</td><td rowspan="2">TT</td><td>CT</td><td rowspan="2">CP</td><td>CL</td></tr>
<tr><td>Antinominant .</td><td>πFL</td><td>πFT</td><td>πCT</td><td>πCL</td></tr>
</table>

<table>
<tr><td>Pythagorean .</td><td rowspan="2">FL</td><td>FP</td><td rowspan="2">FT</td><td>TT</td><td rowspan="2">CT</td><td>CP</td><td rowspan="2">CL</td></tr>
<tr><td>Antinominant .</td><td>πFP</td><td>πTT</td><td>πCP</td></tr>
</table>

We note that the coinciding members form two groups; one of three members, Doh, Ray, Me, and the other of four members, Fah, Soh, Lah, Te, which are separated from each other by an antinominant step at each end (Me : Fah) (Te : Doh) and whose internal members differ by an oscillant.

This discrimination is very important.

The phase relationship between any two terms may also be regarded as compounded from, or resolvable into, two projections in opposite directions, or similar directions unequal in amount.

In opposite directions these appear as located by the respective fractions of the whole cycle; thus the relation Doh : Te on the Pythagorean may be put $T\frac{5}{12}$ $L\frac{7}{12}$

In similar directions, the expression is one of Least Common Multiple, the resultant interval being expressed as a Lead or Lag of phase.

Both these expressions are binary, and may be illustrated by a reference to the aspect in which plane polarised light is regarded in Optics as compounded of dextral and laeval components, or reference may be made to the analogous phenomenon of fluctuation (beats) in physical acoustics.

The phase significance of every relationship appears thus to inhere in a duality, which involves a binary expression.

Of these, species is the one particular experiential discrimination, which may be partialised into its elements of order, viz.:—

Zero order: Pure direction on the pitch range.

First order: Polarity of extensor elements.

Second order: Permutability of determinator elements.

Again, we have recognised such relationships as "orthogonal species," discriminating between Diatonic and "Accidental" elements of the dodecanal cycle.

In actual experience, it is noted that the abstract equations, expressed by

$$FP=CPD$$
$$CP=FPD$$

do not hold.

There is a preponderance of audentity on one side or the other, corresponding roughly to the "Dominant and Recessive" expressions of Mendelism.

We cannot regard, at any instant, a single value as being simultaneously of more than one order of tonality (*i.e.* a Prime being a Determinator); but it appears from the high development of modern harmonic polyphony, that the transitional forms on the boundary may be utilised with such freedom as to approach very near to the identity of dualism. This delineates the region between determinance and chaos in tonal presentation.

The simpler view of tonality, upon which the great tone masters appeared to work, insists upon the maintenance of a definite excess of audentity on either one side or the other.

Thus the tones Me and Doh never fused into homochromal indefinity, although transitions and compounds of numerous and complex types were by no means precluded thereby.

The duality of CP, as dominant of a minor key, leads directly to a homochromal treatment, unless direct or associative reference to just intonation is maintained.

The E.T. value of G can be conventionally regarded on occasion

as either one of two contrasting extremes: viz. diminished R or a just second-order chrome.

This is an example of the complex possibility of tonal material arising from the Symmetrical Tertriad in E.T.

From a fundamental aspect, the Coincidental Prime has an infinity of "orders," which, in the triad, is limited to three.

Vertication, as enhancing a fundamental aspect, thus opens the door to the modern complexities of tonality.

SUBSECTION 3

PHASE ASPECT OF TRIADS

The location and relationships of triads within the matrix and domain may be expressed in phase terms by the projection upon the dodecanal cycle.

Any given triad has three different aspects according to the chrome selected as base.

The primary form has the first-order chrome as base, from which the educt determinator and its permute can be projected as extra-dodecanal elements.

When lines are drawn from the dodecanal member representing the determinator to the base, the resulting triangular figure represents the triad.

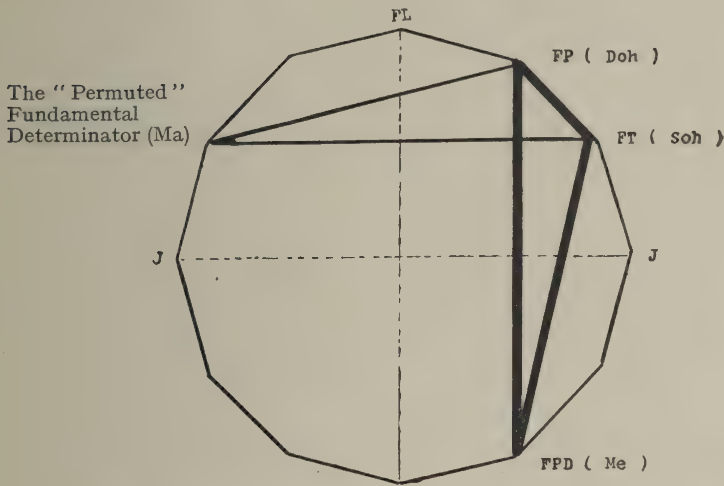
This presents the position and relationship to the eye in a definite manner.

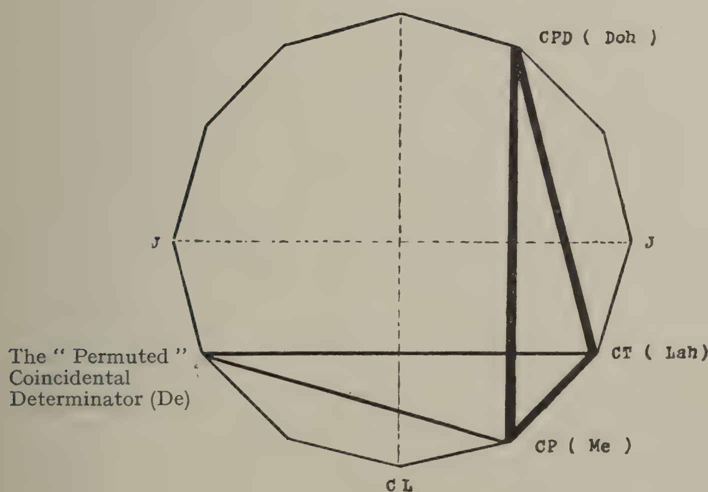
The educt determinator "apex" is in the same hemicycle as the base, while its permuted form is seen to be situate in the "diametrical" hemicycle.

The two principal triads, with their permutes, are shown in the figure. The thick lines represent the original triad of each species on the base (P : T), in the respective concomitant quadrants.

The lines representing the chromal components are seen to be respectively portions of the Dodecanal, Tetragonal, and Trigonal cycles.

The operation of permuting a determinator is represented by the symmetrical displacement of the apex of the triangle from the direction of the original species to its "mirror image" in the dodecagon.


FL
FP (Doh)
The " Permuted " Fundamental Determinator (Ma)
FT (Soh)
J
J
FPD (Me)


CPD (Doh)
J
J
The " Permuted " Coincidental Determinator (De)
CT (Lah)
CP (Me)
C L

Although permutation is notationally equivalent to total commutation (as regards notes) it is seen to be a partial operation.

The triad as a whole may still be considered to be in its original species, only the second-order element having moved.

The importance of this view will be seen when considering chordal developments.

The corresponding operation in the Antinominant Cycle is indicated by the substitution of A for (B or R) as a base.

In this case, the figure no longer represents a triad, so called, but a "Booleian" multiple, a superposition of V upon G; which may be regarded as a triad, one of whose second-order dyads has been folded over upon the other about the determinator as axis. The second-order relationship between D and either $(P+\pi T)$ or $(T+\pi T)$ remains.

The fact that Pythagorean and Antinominant values differ by π at every alternate step, may be noted in connection with the variability of second-order terms since:—

$$V+W/2=Y \text{ and } 3Y=2W,\ 3A=V$$

Consequently $3B-3A=B-A=W/2$.

The operation of permuting an educt determinator of given species into its adduct of the converse species, has the effect of contracting the chrome F(P : D) from G to V and expanding the chrome C(T : D) from V to G (with respective applemental expansion and contraction).

The special symbol used to indicate the permute determinator is suggested by the conventional colour of the keyboard digitals. In the dual concomitance of the "natural" key (FP=Doh, CP=Me) PD, LD, and TD are represented by the white digitals F (Me, Lah, Te) and C (Doh, Soh, Fah) respectively.

Their permutes are P◗, L◗, and T◗, respectively represented by black digitals, *i.e.* by the notes F (Ma, La, Ta), C (De, Se, Fe).

The operation of permutation thus involves the depression of the fundamental, and elevation of the coincidental determinator, by one E.T.A. in pitch.

The symmetrical determinator (the Indeterminator), of theoretical interest only, is conveniently symbolised by Đ.

The digital analogy is unfortunately reversed in the case of the contra-determinators, but the symbolism may also be extended to these values, or rather, to their considerably tempered repre-

sentatives, so that ᗡ stands for the educt and ◖ for the adduct (permuted) contra-determinators respectively.

In Tonic-Solfa, these values are given by:—

	Fundamental.	Coincidental.
LD	Lah	Soh
PD	Me	Doh
TD	Te	Fah
L◗	La–Se	Se–La
P◗	Ma	De
T◗	Ta	Fe
Lᗡ	Ma–Re	De–Ra
Pᗡ	Ta–Le	Fe–Sa
Tᗡ	Fah–Mee	Te–Da
L◖	Ray	Ray
P◖	Lah	Soh
T◖	Me	Doh
LÐ	Lae	Soe
PÐ	Mae	Doe
TÐ	Tae	Fae
Lᗡ̵	Rae	Rae
Pᗡ̵	Lae	Sae
Tᗡ̵	Fae	Dae

SUBSECTION 4

HEMICYCLE AND QUADRANT

The subdivision of the dodecanal cycle into two hemicycles about the diameter $(J-\pi J)$ coincides with the discrimination of the two species F and C.

This leads to an inquiry as to the discrimination of elements when the phase division is "orthogonal" to the $(J-\pi J)$ diameter.

For convenience, this discrimination of the two hemicycles may be referred to as "Ortho-species."

Taking the Pythagorean dodecanal as basis, we note that the dextral hemicycle with its two terminators comprises the diatonic or "natural" elements, while the remaining five components

on the laeval side are excluded, "accidental" or "chromatic" in respect to the nominant of the Cycle.

A convenient distinction may be made by reference to the usual arrangement of the keyboard, where the seven white notes are Dextral and the five black notes are Laeval components of the "Natural" key.

The precise meaning of such terms as "diatonic," "natural," etc., will come under discussion in connection with chordal systems.

At present we are concerned with the aspects of the Septomial about J, and its complementary synonym (the laeval Quinomial) about πJ.

The Septomial appears as an expression of equated Species terms about J. In Zero first-order terms, reading downwards:—

FL : FP : FT : J : CT : CP : CL

It may also be expressed in terms of one species by transforming the "recessive" element into second-order terms:—

FL : FP : FT : J : FLD : FPD : FTD

or

CTD : CPD : CLD : J : CT : CP : CL

The laeval hemicycle may be similarly expressed in diametrical terms:—

πCL : πCP : πCT : πJ : πFT : πFP : πFL

which is equivalent to a reversal of Phase (Zero Commutation) and a diametrical transformation.

The above expressions can be converted into those of the Antinominant Cycle by prefixing π to every odd term, counting J as Zero; π^2 vanishing.

The Fundamental diametricals are conventionally considered as "sharp" and the Coincidental diametricals as "flat" operators, respectively: all being expressible in terms of each other, and equating at πJ (Se, La).

A particular aspect of the Dextral Pythagorean Hemicycle may now be touched upon, viz. the approximation of the components to those of the odd series of Tensor and Laxator.

The nearest approximation is in the case of the Tensor series, as follows:—Σ_1^{13}T =

FT1	Dial Phase	II	CT1	Dial Phase	IV
3	,,	III	3	,,	III
5	,,	VI	5	,,	XII
7	,,	XII	7	,,	VI
9	,,	IV	9	,,	II
11	,,	I	11	,,	V
13	,,	V	13	,,	I

$$CT_3 = FT_3 = J$$

Similarly, we may note:—$\Sigma_1^{13}L =$

FL1	Dial Phase	XII	CL1	Dial Phase	VI
3	,,	I	3	,,	V
5	,,	IV	5	,,	II
7	,,	(X) III	7	,,	(VIII) III
9	,,	II	9	,,	IV
11	,,	VI	11	,,	XII
13	,,	III	13	,,	III

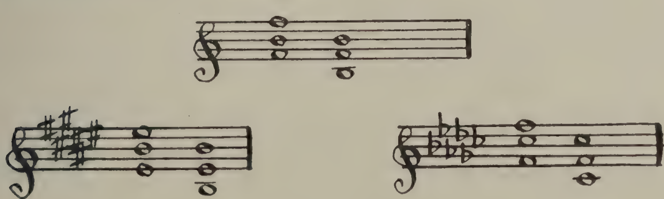
In this case the approximation is less close.

The discrimination of the hemichrome W/2 into augmented R or diminished B is the factor which identifies an arbitrary term as either L or TD (*i.e.* a polar discrimination).

In the antinominant cycle, as in the Pythagorean, six infra steps project to a " diametrical," while six ultra steps from thence " retract " to the achrome of the original nominant.

But in diatonic terms, there is the discrimination between the group containing 2O+2A, and 3O, the first named being " diminished B " and the second " augmented R."

In Staff notation, the discrimination is shown by a rise or fall of the expression:—



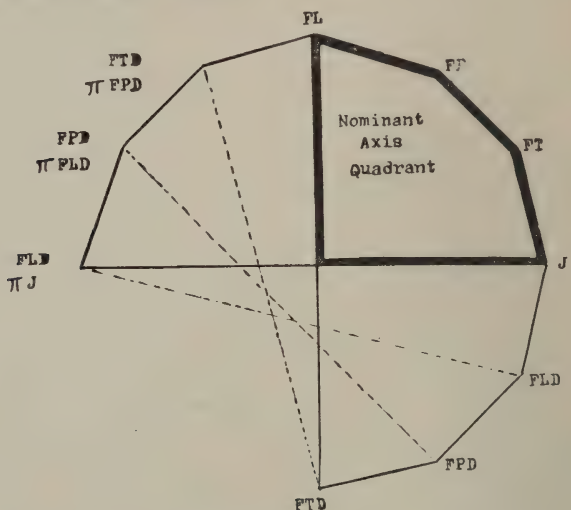
The discrimination appears in just intonation, as primarily between F(L : TD)=C(TD : L).

In the Series it is approximated by:—

$$F(P_5 : {}_7 : {}_{10}) = C_5(P_{10} : {}_7 : {}_5)$$
$$C(P_{10} : {}_7 : {}_5) = F_5(P_5 : {}_7 : {}_{10})$$

With respect to any nominant, π may be reached by the two opposite phase paths, each consisting of six antinominant or first-order chrome steps, or two ultra-second-order chrome steps (V and Y), or three oscillant fluent steps.

The actual use of π, therefore, requires a prescript of Species, Vertication, Polarity, Permutation, Clinearity, etc., according to the Order and Type in which the units of phase are expressed: and these prescripts equate alternately, according to their general representative signs + and −.



The application of one of these operators to a first-order chrome expands R and contracts B into W/2. Consequently, a pair of oppositely directed π operators applied respectively to each terminator contracts B to R and expands R to B.

Similar operations are possible with second-order chromes, but in this case the units are less than an antinominant, and therefore do not appear in the dodecanal system.

The operator in this case would be one which expands V or shrinks G into B/2.

The pair of oppositely directed operators would similarly contract G into V and expand V into G.

It may be noted that the semi-operators $\pi/2$ and $3\pi/2$ correspond to V and Y respectively; and that when applied to a phase quadrant they rotate it through a quarter-cycle.

If a hemicycle be treated in the same manner, the quadrant common to both initial and resultant loci is the Axis of the operation; and if the axis contains first-order elements, the operation "permutes" the determinators by changing educt into adduct, and *vice versa*.

This is conveniently illustrated by the figure (on opposite page) of such an operation about the F quadrant as Axis.

The dotted lines show the permutation of the determinators.

SUBSECTION 5

SUBCYCLES

The quadrinomial antinominant subcycle (period G) is of some special interest in consequence of its intervals corresponding roughly to the dimensions of the Degrees of Tonal Determinance.

A as superior limit of Liminal tempering.

2A as principal fluent.

3A as minimum chrome.

A cycle of whole period B or R is theoretically possible, but its four quadrantal grades are inexpressible in E.T. terms.

(Possibly some of the Persian systems with 16 steps to the octave might be referred to some such basis.)

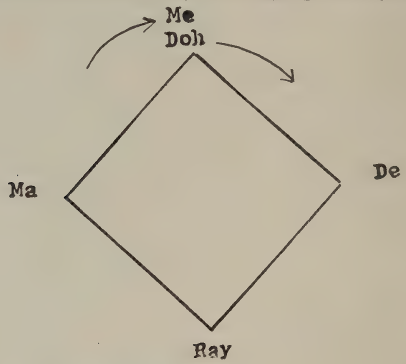
The cycles of G and M are capable of resolution into 4A and 4O respectively.

By means of the alternate diametrical translation, the terms may be put into Pythagorean form.

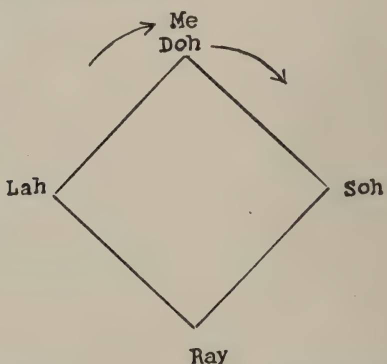
The cycle of G trisects the Octave, there are, therefore, three such groups possible within the Dodecagon.

However, another aspect is prominent. The octave period may be considered as divided into two unequal periods of 4A and 4O.

The respective forms may be plotted graphically as Tetragons:—



Antinominant Fluent forms:—



Tetragonal. Pythagorean Forms.

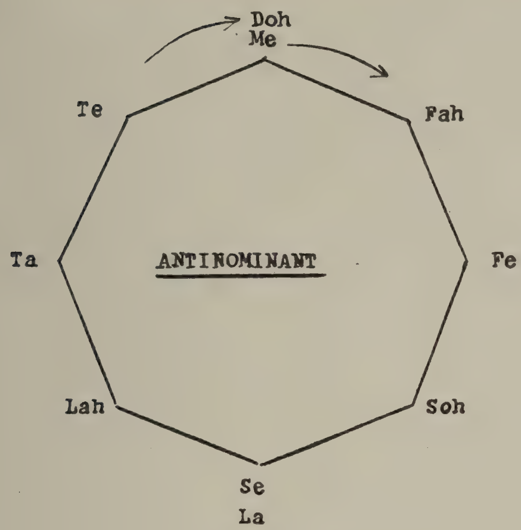
In these diagrams the transformation about the " horizontal axis," by means of the diametrical operator, is seen.

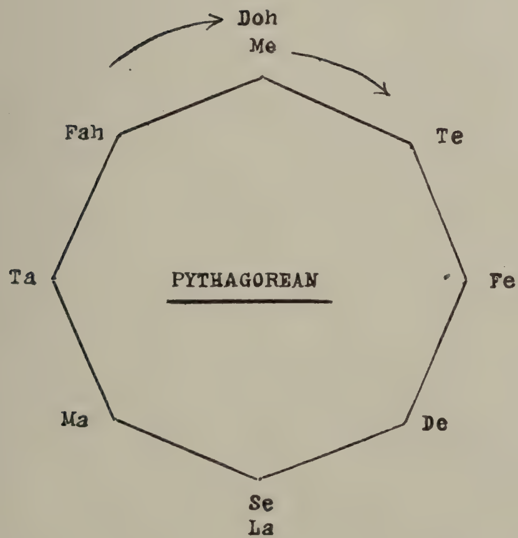
The complementary forms, with respect of the dodecagon, may be plotted as octagonal cycles.

In these diagrams, the alternate relationship of diametricalism is also seen to hold.

The formation of a quasi-cycle upon the somewhat slight recurrent characteristics inhering in Doh and Me is somewhat prominent as an aspect of the dual concomitance expressed by the equation:—

$$\,^{F}_{C}P = \,^{C}_{F}PD$$


Doh
Me
Te
Fah
Ta
ANTINOMINANT
Fe
Lah
Soh
Se
La


Doh
Me
Fah
Te
Ta
PYTHAGOREAN
Fe
Ma
De
Se
La

In this particular subcyclic view, discrimination of species is assumed to vanish, so that we can say that the Determinator marks a recurrence of the Prime.

The diametricals of the origin are J and πJ for the G and M periods respectively.

The cycle may be regarded as made up of two oscillant or binoscillant hemicycles, forming a complete sesqui-cycle or "undulation."

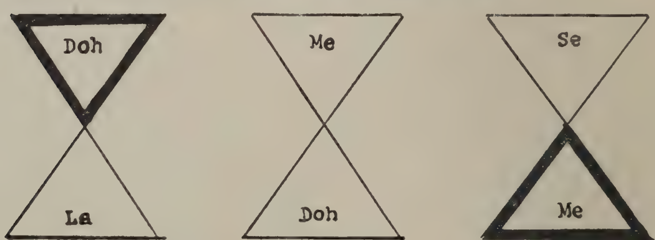
According to the view of Hauptmann, this presents a case in which first-order relationship vanishes through the attainment of second-order chromal unity.

The three trisectants of the Dodecanomial may be considered in Trinomial form, with (P=PD) as Centron; thus appearing in F species:—

Laeval.	Centron.	Dextral.
(La : Ta : Doh)	(Doh : Ray : Me)	(Me : Fe : Se)

and conversely in C species.

Each inheres in two triads, as the figure shows:—



Of these "poles," one is binaxial, the other monaxial, in relationship.

The binaxial relationships are illustrated by the thicker lines, showing the audental differentiation of the Species in the "poles" of the expression.

Hence it is to be noted that the Trisectant aspect shows the arbitrary species associated with the arrangement.

The neutral "species" of the quasi-second-order chrome formed by $\begin{pmatrix}\text{Me} & : \text{La} \\ \text{Se} & : \text{Fa}\end{pmatrix}$ is of the same type as the yoke J.

It actually involves the "tempered" equation 3G=W.

J may thus appear as a Prime of neutral species; and its

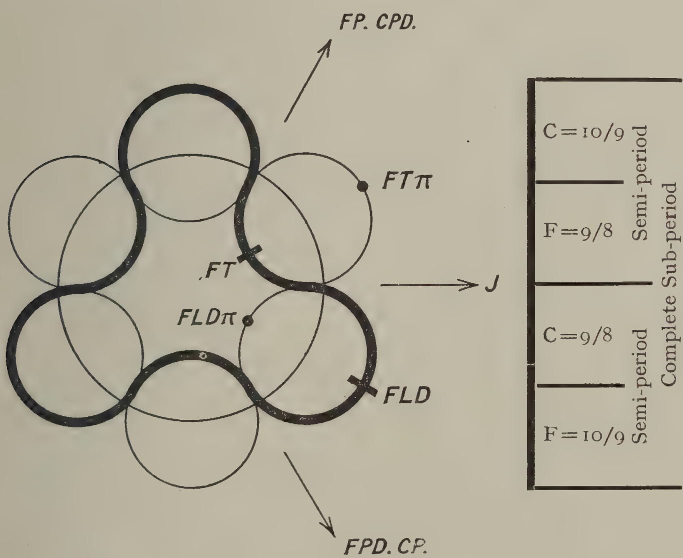
orthogonality to L, which is the maximum of species audentity, confirms this view.

In actual experience, the arbitrary vertication conditions affect the symmetry of these relations; but we are at present only concerned with the abstract aspect due to the hypothetical elimination of Vertication.

The ratio (M : G)=2 corresponds to the frequency relation of the Octave, in a logarithmic or higher order (in the mathematical sense) of values.

The composition of the two types of cycle may be represented by some such diagram as the following.

In this, the Pythagorean values are arbitrarily distinguished by a thick line.



It would indeed be possible to go on multiplying diagrams illustrative of Phase relations *ad infinitum,* and the reader may confidently be left to do this in order to become familiar with the aspects.

It may be noted that J and its diametrical are the "diacentrons" of the subcycles G and M.

In this case, the commatic tempering, which permits the expression of

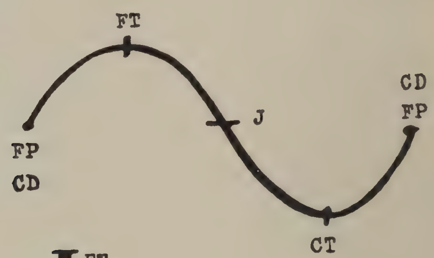
$$P : J :: J : D$$

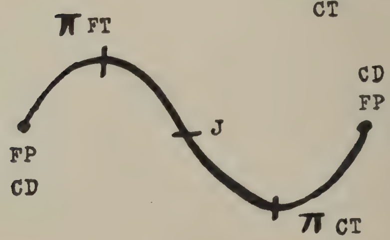
is seen to inhere in the neutrality of the species discrimination between

$$F9/8 : 10/9, C10/9 : 9/8$$

which has been seen to depend directly upon the auditory experience of Tempering.

The trisectional division of W into G and M is paralleled by that of V into A and O; the order of reading determining the species.





Although the trisectional division is not directly resolvable into orthogonal components, it may be regarded as a Hexagon, of a complete period 2W, otherwise as a dodecagon of period 4W.

This, again, involves a specific aspect of the Zero Partial Species, in the form of a definite multiple Achrome being taken as whole period, instead of the unit that has been assumed as such so far.

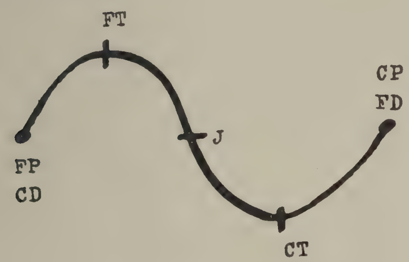
The Tetragonal Cycle (the orthogonal resolutes of any cycle divisible into four parts) may also be illustrated by a Wave diagram, which can represent the composition of two oppositely-proceeding similar progressive waves of semi-amplitude.

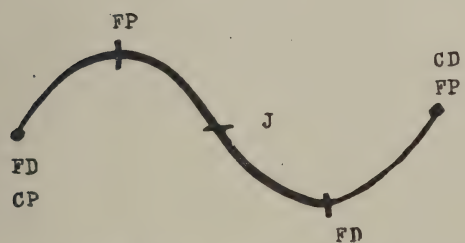
Here, again, the binary or dual inherence of tonal relationship appears.

The Dodecanal Pythagorean and the similar antinominant wave diagrams appear respectively as in diagram on previous page.

Either of these can be "commuted" in chirality or ortho-species, by transforming the diametrical operator, or by reading the Pythagorean as Antinominant Terms, and *vice versa*.

The corresponding Subcyclic "Waves," with periods of G and M, may also be shown:—





The two generating wave trains may be expressed as undulations of Species, Ortho-species, etc.

The oscillant steps represent an experienced undulation approximating to Con- and Dis-cordance when the intervals are summed.

The practical predominance of the G subcycle in experience arises from the fact that the whole period approximately corresponds to the inferior limit of concordance (Mayer's theory).

The overlap of parasyntonic tracts, in the middle pitch region, is practically negligible at G; but πCT and πPT form pronounced discords with J.

These values are the quadrates of the subcycle.

Three-quarters of the G cycle is equal to V, which stands on the boundary between chromal concordance and fluent discordance.

These effects are sufficiently evident to support the view as to a quasi-recurrent character inhering in the subcycle period G.

SUBSECTION 6

MULTI-DODECANALS

When three adjacent triads on the dodecanal cycle are written in line, thus:—

$$\text{L : LD : LT} \qquad \text{T : TD : TT}$$
$$\text{P : PD : T}$$

it is seen that two "recessive" (concomitant) triads are formulated between them, viz.:—

$$\text{LD} : \frac{\text{LT}}{\text{P}} : \text{PD and PD : T : TD}$$

If these chords be regarded as halfway stages in a progression from the central triad to either of its "polars" they may be termed "Seminomials," which can be interpolated in the dodecanal cycle.

In the example quoted above, we have what may be known as the Semi-Laxator and Semi-Tensor triads.

Although actually compound chords, these have the same appearance as simple triads, and if they be included in the cycle, we obtain a Bi-dodecanal cycle of twenty-four terms.

The Antinominant Cycle similarly permits of the interpolation of two intermediate stages between each nomial, but in this case the conditions are somewhat complex, owing to the non-axial progression; or rather, as will be seen later, to the fact that the direct progression is really a part of a more composite chordal schema of Tetrads.

The principal object in mentioning the "Seminomial" theory at this stage, is the provision of an adequate Phase terminology for these "Commute" intermediate Triads.

It must not be understood that the relationship of these chords is in any way compatible with those of the direct Pythagorean cycle, or that only the limited relationship of phase applies;

otherwise the criticism that the employment of these "Seminomial" Triads is different in many cases to the suggested phase nomination might be maintained, with detriment to the convenience of the symbolism.

The seminomial appears as an intermediate stage of chordal progression when the whole of the notes do not move (or change) simultaneously.

The process of splitting a progression up into stages may of course be effected in many different ways.

Of these, only a few of the predominant cases are of particular interest at present, among which the above-mentioned case is evident.

In a similar manner, the "Permuted" Triads of a specified matrix may also be regarded as "seminomials," or intermediate stages.

Thus, on each side of the Prime triad we may have:—

Laxator.	Semi-laxator.	Centron.	Semi-tensor.	Tensor.
LD	L◗	T	T	T
L	L	D	◗	TT
P	P	P	P	TD

The aspect in which P_{19} is sometimes regarded as an intermediate stage between P_9 and P_{10} is seen to be rational in the above case.

The case is an incomplete stage of the whole progressional aspect, in which the relationship of Determinator and Contradeterminator is involved (the criterion being the appearance of W/2 between $\frac{\text{D}}{◗} : \frac{ᗡ}{◖}$).

The real function of the permuted determinator is that of a Commute "Leading Note."

The direction of a "leading note," a second-order term, according to its partial species, is allied to that of the Pythagorean first-order cyclic locus direction.

This is a case of concurrence between chordal status and progressional direction, which matters are studied in detail under the heading of Progression.

CHAPTER IX

SUBSECTION 1

THE MATRIX

One of the most evident facts about tonality (manifested quite as much by those systems of practical working which ignore it, as by those which are based upon it) is the possibility of nominating any element with respect to a locus on the pitch range; the locus being capable of expression relative to other loci, as well as statement in quasi-arbitrary terms of the auditory limits and pitch-range characteristics.

Up to the extent to which this principle holds, every tone, chrome, chord, progression, etc., can be stated as part of a system whose locus is expressible as an entity. This is the idea of " being in a key."

Such an aggregation of elements (whether simple tones or compounds) may be known as a Matrix.

The Locus of a Matrix may wander over a region which may be called its Domain.

Both these aggregates are in reality infinitely extended; but, by the principle of the E.T. due to the liminal equation of approximating values, it is possible to represent the tonal formulation by means of a comparatively few notes.

The Locus of a matrix is expressed by the pitch position of one of its terms which acts as a Nominant basis.

This term is (theoretically, at least) the component which behaves as the centre of symmetry, somewhat in the same way that the centroid in mechanics represents the locus of a mass.

The particular system of tones, etc., constituting a matrix may be arrived at by following up various chains of relationship to their limits. Each system constitutes a special type of matrix of greater or lesser audentity relative to any other system.

When, however, several different aspects, each of considerable individual importance, tend to conform to a common system (which may be variable in its subordinate details), we then arrive at the idea of a General Matrix.

Such is the idea behind the conventional expression of " Key."

"Key" is a term used somewhat loosely in the musical sense to denote an aggregation comprising tones, intervals, triads, chords, progressions, scales, etc.

The great point about the idea is that the aggregation has a definite locus, which can be moved in pitch by the operation of modulation (achromes not counted).

The region of key-relationships over which the locus moves constitutes its "Domain"; and the locus is referred to a representative tone, which may either be the "Centroid" of symmetry or an important member, which then is referred to as the Keynote or Tonic.

The notion of "Key," although possibly adequate for all practical purposes, is too vague, and involves too many nominalistic disputations to be of much use in an abstract examination.

Hence the use of such special terms as "Matrix," "Locus," "Conservation," "Translation," etc.

For a somewhat fanciful illustration, the idea of a Matrix may be compared to a solar system having a locus in a universe of similar systems, which comprises a Sun (the Centron) and its attendant planets with their satellites, consisting of concentric shells of solid, liquid, and gaseous matter, the whole being held together by physical forces.

The available tone members may fall into the two classes:—

(1) Included, or Conserved.

(2) Excluded, or Translatory.

Among these, certain members may be alternative, *i.e.* capable of being exchanged for others.

There are three prominent aspects or types of the general matrix, which may be arranged in order of audentity:—

(1) Tertriadal.

(2) Hemicyclic.

(3) Seriopolar.

There are two principal aggregating agencies by which a Matrix may be formulated:—

(1) Serial Coherence.

(2) Cyclic Adherence (by axial or phonic extension).

The criterion of limitation is the univalent nominance of all terms.

Plurivalence implies a multiplicity of bases for reference, and thus a state of translation between definite loci.

The triad, as a limited portion of a series, represents the expansion of a single note in pitch, the whole being regarded as a Nomial, any of whose adherent relationships may be continued to form a closed cycle; thus being expressible in phase terms.

There will obviously be as many systems of matrical extension as there are cycles. The principal cases are the general Dodecanal cycles in the two extremes of the Pythagorean (coherent and chromal) and the Antinominant (adherent or phonal), since the number 12 is the least common multiple of the included frequency relations, and the least sum of the pitch grouping integrals.

The symmetrical plan of distribution necessitates the recognition of a Locus (Centroid) and the persistence of the non-centron nominality of each component when presented apart from the centron.

This implies some way whereby a given member on an otherwise unlocated cycle can be assigned to the position of zero nominant, or Centron, while from it radiate its dextral and laeval poles.

Owing to the cyclic relationship, the poles diverge, but curve round until they meet at the member diametrical to the centron, the tangents passing through a complete cycle of mutual relationship.

In the cycle of tone-nomials, such a located system is given by a minimum of three terms called a Trinomial.

This consists of a centron, its dextral and laeval poles.

The relationship may be regarded as derived from two terms and a relation, to which is added a third term, standing in the same relation to the centron as the latter does to its projectory pole.

The general form is given by three adjacent terms of a cycle:—

Pythagorean	$L : P : T$
Antinominant	$\pi L : P : \pi T$

By extension of this process we obtain further groupings, the method being as follows:—

Unomial	P
Trinomial	$L : P : T$
Quinomial	$L^{II} : L : P : T : T^{II}$
Septomial	$L^{III} : L^{II} : L : P : T : T^{II} : T^{III}$
Nonomial	$L^{IV} : L^{III} : L^{II} : L : P : T : T^{II} : T^{III} : T^{IV}$
Ondecanomial	$L^{V} : L^{IV} : L^{III} : L^{II} : L : P : T : T^{II} : T^{III} : T^{IV} : T^{V}$

The limit is reached at the Ondecanomial, the remaining (excluded) nomial being the ambiguous Diametrical Centron (Diacentron) which is, in E.T., equivalent to the Sexalaxator and Sexatensor, viz.:—

$$\pi P = L^{VI} = T^{VI}$$

The general constitution of a system appears to be that of a Centron as a " Core," and successive " Envelopes " consisting of its poles.

The symbol E may stand for the envelopes, and by the adoption of the index system the centron may be regarded as the Zero-envelope E^0; the unit E^1 is the first pair of poles, and so on.

Passing now to the consideration of the chrome system; as before, it is evident that the most general is the Dodecanal, given in extreme form by the Pythagorean (Grade Unit B or R) and Antinominant (Grade Unit A) respectively.

Putting our examples into Pythagorean form (from which the Antinominant expressions can easily be derived) we have as Trinomial:—

Laeval Pole.	Centron.	Dextral Pole.
(L : P)	(P : T)	(T : TT)

with corresponding extension as with tone nomials, limited at the ondecanomial, and always excluding the ambiguous bivalent diametrical nomial:—

$$B^{\pm 6},\quad R^{\pm 6},\quad A^{\pm 6}$$

These cycles can be put into the form:—

$$B^{-5}\ B^{-4}\ B^{-3}\ B^{-2}\ B^{-1}\ B^{0}\ B^{1}\ B^{2}\ B^{3}\ B^{4}\ B^{5}$$
$$R^{-5}\ R^{-4}\ R^{-3}\ R^{-2}\ R^{-1}\ R^{0}\ R^{1}\ R^{2}\ R^{3}\ R^{4}\ R^{5}$$
$$A^{-5}\ A^{-4}\ A^{-3}\ A_{-}^{2}\ A^{-1}\ A^{0}\ A^{1}\ A^{2}\ A^{3}\ A^{4}\ A^{5}$$

The Trichromal or Trifluent group is particularly noticeable as being identical with a tetranomial of tones, *i.e.* a quadrant of the Dodecanal cycle.

The principles of tangent relationship have been discussed under the heading of Phase, but it may now be recalled that the " tangents " of the polar nomials are:—

At the Centron	Colinear divergent
At the Quadrant	Orthogonal divergent

At the Semiperiod	Parallel; oppositely directed
At the Anti-quadrant	Orthogonal convergent
At the Diametrical	Colinear convergent

In the case of the chrome and antinominant trinomial it is evident that dyads formed by the tone members L : TT or πL : TT are equal (in E.T.) to a chrome of second order.

By interpolating determinators, the Pythagorean B Trichrome becomes a Tertriad.

This is the most important matrical system in tonality, since it constitutes the basis of chordal nominance.

By interpolating contradeterminators in a similar R Trichrome, a pseudo-tertriad is evolved having some analogous properties.

It is not possible to interpolate any E.T. tone in the Antinominant cycle, but chromes, triads, and chords generally, can be substituted for its tone nomials, and thus forms comparable with those of the Pythagorean system may arise.

SUBSECTION 2

TERTRIAD AND HEPTAD

A Tertriad has been defined as a group consisting of a Trinomial of Triads, and the aspect may be variously regarded as:—

(1) A Trinomial of Triads.
(2) A "Triad" of Trinomial Tones.
(3) A Trichromal, with interpolated Determinators.

These may be written in a general group, in which the respective aspects read horizontally and vertically, and with a typographical differentiation of second-order terms.

There are as many methods of aggregation as there are disposable variables, but, for purposes of notation, it is convenient to regard the group as the sum of the Centron and Polar triads.

The Trichromal aspect (Homochrome of three "B's") has the advantage of exhibiting the phase quadrant (Quadrinomial of tones) and the obvious limitation of univalence consequent upon the approximation of the extreme members to an independent relationship of second order.

It also permits the interpolated determinators to be regarded as independent of the species of the triad, which is an important aspect, and a convenience in comparative examination.

The Tertriad is a group of seven tones, and its claim to be regarded as a matrix rests upon the fact that the Polar triads are differentiated from the Centron.

It is an obvious experience that it is possible to present triads of the " Subdominant " (FL) and " Dominant " (FT) in a " Key " (conserved matrix) without giving rise to an effect of modulation into the keys of which those triads are the tonics.

This is seen to be due to the fact that the two " polar " triads are experienced as parts of a general " discord " comprising both in its complete form.

In general, the test of conservation is to convert the dominant triad into a dominant seventh, and the subdominant triad into a " chord of the added sixth." If this can be done without apparent effect on the sense of key, the triads in question are conserved.

This process is, in fact, simply the balancing of polarity about the centron by the addition of members from the opposite pole.

The differentiation of polar triads from the Centron is capable of being demonstrated in another way.

The Prime and Tensor are tones common to the L polar and Centron, T polar and Centron respectively.

Now the Laxator is the " Commute " Species Tensor, with respect to the Prime, so that the Trichrome may be regarded as of three first-order chromes, having the centron neutral in species, the dextral polar projected fundamentally and the laeval polar coincidentally.

We may, therefore, take the Dyad (P : T) as Centron, and project a fundamental series upward from T, and a coincidental series downward from P.

The process may be continued to any number of members.

1 : 3 presents the Trichrome.
1 : 3 : 5 presents the Tertriad (Prime Indeterminator eliminated).
1 : 3 : 5 : 7 presents a mutually determined Tertriad.

Representing the tones in Tonic-Solfa, we obtain symmetrical grouping about the Centron Blue as follows:—

	Fundamental Example.		Coincidental Example.
13	Me–Ma		Se–Soh
11	Doh–De		Lah–Le
9	Lah		Fe
7	Fah–Me		Ray–Ra
3	Ray		Te
5	Te		Se
1	SOH DOH	CENTRON	ME LAH
5	La		Fah
3	Fah		Ray
7	Ray–Re		Te–Doh
9	Ta		Soh
11	Soh–Sa		Me–Ma
13	Ma–Me		Doh–De

The quasi-coincidence of the achromes of the opposite polar series members is observed. For instance, it is noted that $C_7=F_3$ and *vice versa*.

Again, F_{11} of T equals C_{11} of P, giving rise to quasi axes, between which the four notes forming the Envelope Tetrad and their two alternative determinator forms should be noted.

Among all possible varieties of Tertriad due to the disposability of the determinators, two forms stand prominently forward. These are:—

(1) The Monocyclic form; consisting of similar triads upon a trinomial of tones (*i.e.* a pattern triad repeated cyclically three times).

In this case, the determinators are all of one and the same species, viz. that of the centron triad as pattern.

Commutation of the Tertriad permutes the three determinators, the quadrinomial of first zero-order tones being axial.

(2) The Symmetrical form; in which the determinators of each triad agree with the local species of that triad, irrespective of the centron.

The Centron triad, in this case, has no partial species, being equated between its two poles; its determinator is therefore ambiguous and settled by external or arbitrary nomination, the

actual centre of symmetry being located at the "Indeterminator" which equally bisects the prime B.

The Tensor determinator, as projector, accords with the species nomination of the group.

Since $L = T^{-1}$, the laxator determinator is of converse species to that of the Tensor.

We thus obtain, for a nominated F species:—

TD is fundamental, LD is coincidental.

With C species nomination, these relations are reversed.

The tones of the monocyclic form of Tertriad (*i.e.* that with similar determinators) constitute the so-called Natural Scale, from which the Major Mode and the Melodic Declinear (Æolian) Minor Mode are delineated by selection of terminals.

This form of the tertriadal type is known as the Hemicyclic because of its identity with another important aspect of the seven-toned matrix.

The symmetrical form of Tertriad, with a coincidental prime determinator, and consequent coincidental nominance, constitutes the so-called "harmonic" form of the Tonic and Relative Minor Scales.

Both groups are seen to be actually Composite in constitution, since they involve a difference of partial species among components of different order in the same triad.

The Hemicyclic form presents a laxator determinator oppositely directed in species to that of the laxator, and the group is thus quite as radically "unnatural" as the Symmetrical form, in which the polar determinators are opposed; in spite of the latter's apparent notational heterogeneity as compared with the former.

The matrical implication of the Symmetric form is evident from the quasi-equation presented by certain series members of the poles.

To go no further along the series than the Contradeterminators, it is seen that:—

TF(P : D : T : Ɑ) is the "Dominant Seventh."

PC(P : D : T : Ɑ) is the "Chord of the Added Sixth" with a "minor" determinator (F nomination).

The poles are differentiated from each other, and mutually from the centron, by the fact that their triads appear in the

matrix as part of the above-mentioned Tetrads. The effect is therefore towards " centring " the centron triad, and consequently the locus of the matrical group is definitely determined both with respect to the domain of possible translation, as well as the position on the range of pitch.

This is Conservation of Matrical Locus.

The same principle is evident in the Hemicyclic form of Tertriad, where, in E.T.:—

$$\mathrm{T(P : D : T) + L = T : TD : TT : T}\text{ᗡ}$$
$$\mathrm{L(P : D : T) + TT = L : TD : LT : L}\text{◖}$$

By writing L in the first equation as PT^{-1}, the symmetry of the arrangement is somewhat more evident.

These two chords are the Radical Polar Tetrads (conveniently symbolised by squares inscribed with the particular polar symbol), and it is also to be noted that the two forms are coupled by the equation:—

$$\mathrm{L : L}\text{◗}\mathrm{ : LT : L}\text{◖}\mathrm{ = PC(T : D : P : }\text{ᗡ}\mathrm{)}$$

A purely symmetrical " Tertriad," or homochrome of six B/2 hemichromes, may be formed by interpolating three " indeterminators " in the Trinomial of B.

This is of little practical interest beyond a theoretical formulation, but the indeterminator represents an optional ambiguity of second-order species, so that the Tertriad appears:—

$$\frac{\mathrm{F}}{\mathrm{C}}\mathrm{(L : L}^{\mathrm{D}}_{\text{◗}}\mathrm{ : LT) : (P : P}^{\mathrm{D}}_{\text{◗}}\mathrm{ : T) : (T : T}^{\mathrm{D}}_{\text{◗}}\mathrm{ : TT)}$$

The permutable independence of each triad is thus evident.

The real " centroid " of the Tertriadal Matrix is the Prime Indeterminator, optionally represented by either species of determinator.

Since the triads are regarded as coherent expansions of the prime tones in the Trinomial, it is convenient to take the Centron Prime as the representative nominator of matrical locus.

As discussed later on, in practical treatment we have to deal with modal exchangeability, rather than the theoretical conversity of species.

Consequently it is found convenient to nominate the common Tetranomial of tones from one of the two possible Primes. The Fundamental, owing to Vertication, is most prominent, and thus

FP and CT are regarded as the respective "keynotes" in practice.

This, of course, does not affect the acceptance of a theoretical centroid based upon symmetrical considerations.

The symmetry of the Tertriad is most obvious upon writing L as T^{-1}, and ᗡ as D^{-1}. This gives, for the general matrical expression:—

$$N^{\pm 1}\left\{T^{-1}(P : D^{\pm 1} : T) : P(P : D^{\pm 1} : T) : T^{\pm 1}(P : D^{\pm 1} : T)\right\}$$

This symbolises commutability of each order by the sign of the index.

Although of somewhat an extreme form, the seven notes of the Tertriad may appear in one chord, so that the group may be named the Heptad.

But this aspect implies univalence of species, and is more apparent in another type of seven-toned matrix, viz. the Serio-polar.

The independent permutability of the three second-order tones permits the formation of six different varieties of tertriad, amongst which are the two specially prominent cases:—

(1) Species of second-order tones alike, all being either (*a*) nominantal, or (*b*) commute, with respect to the zero order or total species, viz.:—

$$\begin{array}{l} F \\ C \end{array}\left\{\begin{array}{l} (T^{-1} : D \quad : P) : (P : D \quad : T) : (T : D \quad : TT) \\ (T^{-1} : D^{-1} : P) : (P : D^{-1} : T) : (T : D^{-1} : TT) \end{array}\right\}$$

(2) Species of each second-order tone agreeable with its own triad, independent of any other.

This gives rise to Composite Tertriads, composed of uni-specified triads, there being the two forms:—

$$\begin{array}{l} F \\ C \end{array}\left\{\begin{array}{l} (T^{-1} : D^{-1} : P) : (P : D \quad : T) : (T : T\dot{D} : TT) \\ (T^{-1} : D^{-1} : P) : (P : D^{-1} : T) : (T : TD : TT) \end{array}\right\}$$

In Tonic-Solfa terms, these values are given by:—

F Species (fah : la : doh) : (doh : me : soh) : (soh : te : ray)
C Species (te : se : me) : (me : doh : lah) : (lah : fah : ray)
F Species (fah : la : doh) : (doh : ma : soh) : (soh : te : ray)
C Species (te : se : me) : (me : de : lah) : (lah : fah : ray)

Conventionally, the second and third "rows" of terms present the diatonic notes of the modes known as the "Relative"

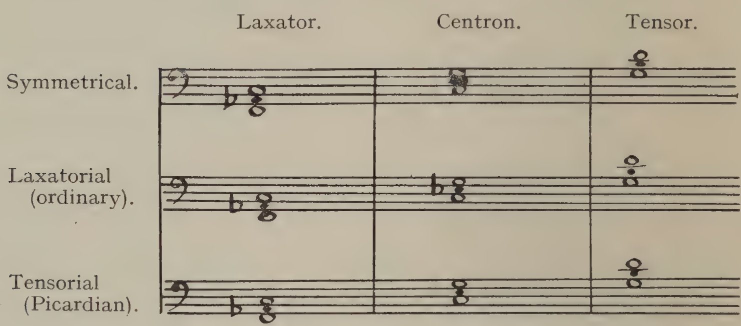
and " Tonic " minor scales respectively to any nominated major key.

The first and fourth rows present modes which have as yet no definite name, but which are used considerably in practice.

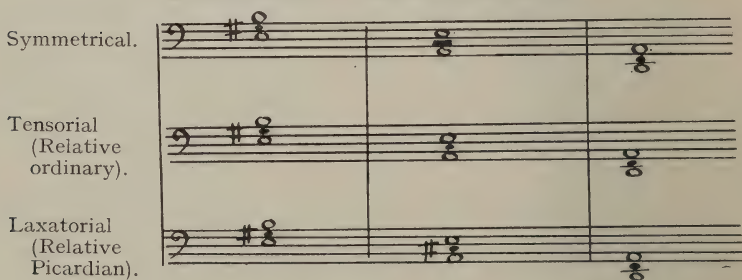
They could be called the corresponding " Picardian " forms, because of the " Tierce de Picardie " in the Prime Triad, which is familiar in final cadences of minor tunes.

The name " Hungarian " has also been suggested.

FUNDAMENTAL SPECIES DERIVATION



COINCIDENTAL SPECIES DERIVATION



Together with any definite type, its reverse must also be considered.

The type in which the species of the second- and zero-order components is identical presents the totally permuted forms of either species.

The component triads in this case are apparently commuted.

They correspond notationally to the " Tonic Minor " or " Tonic Major " of a given Mode, and it is observed that the nominating quadrant or Quadrinomial of first-order members L : P : T : J is axial, so that the permuted form may be regarded as a particular (second-order partial commutation) form of the original species.

The rationale of this apparently topsy-turvy view will be apparent when the progressional aspect is considered, the idea being to separately regard the flux of determinators and the axial persistence of the quadrinomial.

The reverse form of the second type presents:—

$$\left.\begin{matrix}F\\C\end{matrix}\right\} (T^{-1} : D : P) : (P : \frac{D}{D^{-1}} : T) : (T : TD^{-1} : TT)$$

The Tonic-Solfa values are:—

F (fah : lah : doh) : (doh : $\frac{\text{me}}{\text{ma}}$: soh) : (soh : ta : ray)

C (te : soh : me) : (me : $\frac{\text{doh}}{\text{de}}$: lah) : (lah : fe : ray)

These cases are somewhat infrequent in experience, but are theoretically interesting, as being approximated (with PD^{-1}) by the Laxator Series (1 to 13).

SUBSECTION 3

THE TRI-HEPTAD

On repetition " in cascade " of the operation by which the Tertriad was formed, a further stage of tonal development is opened out.

Instead of the triad as a nomial, we may take the Tertriad or Heptad, and the consequent Trinomial presents a group which may be known as the Tri-heptad.

This group is composed of three Tertriads, whose centrons are the respective triads of the Laxator, Prime, and Tensor.

The group thus comprises nine triads, but since four of these may be alike, the group can be reduced to five triads, identical with the Quintriad (Quinomial of triads on adjacent members of the Dodecanal cycle).

It is obvious that such a process of expansion could be carried on *ad infinitum*, presenting, as the next stage, the Ter-tri-heptad, and so on.

In Just Intonation this unlimited expansion, with independent new values, is indeed possible.

But the E.T. system restricts the number of disposable "representative notes" to the economic dodecanal; hence the extension becomes cyclical, and is limited to the Domain.

There is yet another basic method of extension to be considered.

The idea of the Trinomial may be applied to any elementary group of note, triad, heptad, etc., as in the foregoing case.

The Tertriad opens the avenue of extension to the Quintriad, Septriad, Nonatriad, etc., symmetrically disposed about a given centron member which nominates and locates the group.

This method of extension gives rise to groups identical with those arrived at by the previous method.

As an independent aspect due consideration will be given to it.

Apart from the obvious limitation to the E.T. Dodecanal, the matrix presents certain conserved characteristics restricting the grouping to seven tones, which may be generally known as the Septomial.

The Trinomial contains one zero-order tone, and two of first order.

The order of the seven tones of the tertriad is as follows:—

One Zero Order . . .	P
Two First Order . . .	T and L
One First-First Order (Third?) .	TT
Three Second Order . .	PD, TD, and LD

The Tri-heptad is a dodecanomial of one zero, two first, two first-first (third), two first-first-first (fifth), and five second-order components.

It thus comprises twelve independent elements, which, in some of the types, achromatically and liminally coincide, thus reducing the actual number of possible tones to the twelve of the E.T. Domain.

This forms the practical limit of Tonal extension.

The redundant components in the grouping are the Axes

of connection; their function in progression will be considered later.

The Tri-heptad takes the following general form, in which for definity the Triads read horizontally from left to right, and the Trinomials vertically, the centrons being located in the middle row.

The Determinators are independently permutable.

$$\begin{matrix} F \\ C \end{matrix}\left\{\begin{array}{l} (LL : LLD : L) : (L : LD : P) : (P : PD : PT) \\ (L : LD : P) : (P : PD : T) : (T : TD : TT) \\ (P : PD : T) : (T : TD : TT) : (TT : TTD : TTT) \end{array}\right\}$$

Of the new members, in E.T., LLD = TT, and TTT = LD. Again, LL = T◗, and TTD = CT◗, when CP = FPD.

The Tri-heptad allows the twelve tones of the E.T. Domain to be expressed in related terms.

Reading outward from P by Antinominant steps in the direction of the species in pitch, we have:—

	Nominant.	Commute Equivalent.
1.	P = LT = TL	PD
2.	LL◗	P◗
3.	TT = LLD	TT = LLD
4.	P◗	LL◗
5.	PD	P
6.	L = LLT	TD
7.	πP = TTD	T◗
8.	T = PT	LD
9.	L◗	L◗
10.	LD = TTT	T = PT
11.	T◗ = LL	πP = TTD
12.	TD	L = PL = LLT

The Aggregation of the same elements as a Quintriad presents the group as a single row, the general form with optional determinator species being:—

	LAEVAL.		CENTRON.	DEXTRAL.	
	Secondary Polar.	Primary Polar.		Primary Polar.	Secondary Polar.
$\begin{matrix} F \\ C \end{matrix}\Big\{$	(LL : LLD : L) :	(L : LD : P) :	(P : PD : T) :	(T : TD : TT) :	(TT : TTD : TTT) $\Big\}$
F Phase	XI	XII	I	II	III
C Phase	VII	VI	V	IV	III

Septriads, Nonatriads, and so on, may be carried around the Dodecanal Cycle; the nomial whose polarity is indifferentiate is the diametrical centron, and this marks the limit of the extensional process.

It may be noted that the Prime tones of the extreme polar triads achromatically form the following intervals:—

Group.	Interval.		Order.
Triad	P : P	Z	Zero
Tertriad	L : T	O	Third
Quintriad	LL : TT	G	Second
Septriad	LLL : TTT	W/2	Pseudo-first
Nonatriad	LLLL : TTTT	M	Second
Ondecatriad	LLLLL : TTTTT	O	Third
Tridecatriad	LLLLLL : TTTTTT	Z	Zero

The limitation principle is here evident, and it is also seen that LL : TTD forms a second-order interval, the tones being respectively diametrical to PD and P.

The formation of "Pythagorean" second-order chromes by commatic approximation has always been recognised as an important limiting principle, since the two orders of interval in the triad are thus equated by E.T. within the Achrome.

It may be noted that the extreme strings of the violin, viola, and 'cello are in second-order relationship, differing from the pure achrome interval by a Comma (81 : 80).

The Tri-heptad comprises five actual Triads, of which the Centron (that of the Prime) occurs three times, those of the Poles L and T twice, and those of the Bi-laxator and Bi-tensor but once.

Grouping the symmetry of Species into the variabilities of Coincidental, Indeterminate, and Fundamental, it is seen that the species expressions of the respective triadal members appear (on the F Basis) as:—

Triads.			
Bi-tensor			F
Tensor		F	(FC)
Prime	F	(FC)	C
Laxator	(FC)	C	
Bi-laxator	C		

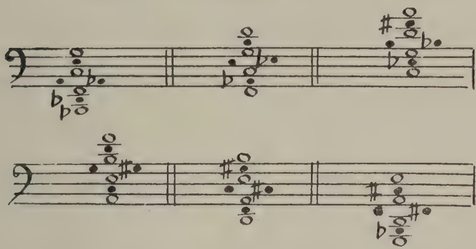
In Staff notation, these appear thus:—

TERTRIADS

L pole. Centron. T pole.

F Species.

C Species.



In the above expressions, the Pythagorean elements (first-order chromal relations) are indicated by open notes, and the second-order terms by closed black notes, whose signature co-efficients indicate the variability of the determinator according to the locus of the Tertriadal " Matrix " in which it occurs.

The variability of the Centron Triad is complete; those of the Tensor and Laxator are biassed or " permuted " in a specific direction; while the two extreme Triads, *i.e.* those of the Bi-laxator and Bi-tensor, are determinate in species.

We may, therefore, regard the symmetric form as the most general expression of Chordal Matrix, out of which two practical types are evolved, viz. that corresponding to the Septomial or Pythagorean Hemicycle and the composite species derived from the Symmetrical Type, from which the various extant forms of Minor mode are selected.

The nomination of species in the dual variability of the Tri-heptad inheres in the discrimination of:—

$$\pi \begin{Bmatrix} \mathrm{FP} = \mathrm{CPD} \\ \mathrm{CP} = \mathrm{FPD} \end{Bmatrix}$$

i.e. in the audental discrimination of the Order of the tones, according to the approximative equations:—

$$\mathrm{LL} = \mathrm{TTDD}$$
$$\mathrm{TTD} = (\mathrm{LL}/\mathrm{T}) \times \mathrm{D}$$

A point to be noted is the " polarisation " of the diametrical elements appearing in the Tertriad.

TTD only appears on the Tensor side, *i.e.* as Fe : Doh, and the E.T. equivalent " Sa " does not show in this univalent term.

Again, LLD only appears on the Laxator side, although in the dual concomitance this term also appears as the "commute" P◗.

The forms of Tri-heptad, in which the species of each determinator corresponds to that of the triad in which it inheres, may now be given:—

TRI-HEPTAD

comprising Triads of:—

Order.	Laxator.			Prime.			Tensor.		
First	L	: P	: T	P	: T	: TT	T	: TT	: TTT
Second	LL◗	: LĐ	: PD	L◗	: PĐ	: TD	P◗	: TĐ	: TTD
Zero	LL	: L	: P	L	: P	: T	P	: T	: TT

The variation of Polarity is to be noted.

The Tervalent Prime constitutes the Centron, while the polar terms are bivalent, and "biassed."

The bilaxator and bitensor are univalent.

This corresponds with the permutable variability of the determinators:—

PD is tervalent in D, Đ, ◗.

LD and TD are bivalent, according to polarity, D, ◗.

LLD and TTD are univalent.

(Đ may be considered as $D^{\sqrt{-1}}$)

The question now arises as to the experienced association of the Symmetrical Matrix with a determinate Species.

This, however, leads to the inquiry as to actual or notational occurrence of such harmonic associations in practice, and is consequently deferred until chordance be discussed.

SUBSECTION 4

THE HEMICYCLIC MATRIX

The idea of a conserved group of tones comprising two species is suggested by the possibility of aggregating a set of values which are dual in species, and which can be derived from the E.T. Dodecanal cycle (in which the prime determinator equals its commatic analogue, the quadritensor).

It is necessary to form very clear conceptions regarding the univalence of a Matrix which embraces a duality of species.

Obviously, both species cannot be preponderant at one time; one mutually excludes the other.

Consequently, the dual values of each component may be discriminated into Nominant and Commute, or, as discussed further on, the aspects can be regarded as respectively Predominant and Recessive.

In practice, one is familiar with this comprehension of duality in the same group. Thus a major and its relative minor keys are comprised under the same signature; a major and its tonic minor keys are nominated by the same prime and tetranomial.

The diatonic "natural" scale serves for the major mode and the declinear melodic form of minor.

Its reciprocal symmetry about TT (and diametrical J) is obvious.

The digitals of the keyboard afford a convenient illustration of the foregoing idea.

It will be noted that the Pythagorean Cycle as arbitrarily arranged on the clock dial (FP as unit member) presents all the white notes on the right-hand half plus the two terminators of the vertical diameter, and the five "black" notes are found on the left-hand half. This is merely a coincidence (in the particular "natural" key), but it favours recollection of the arrangement.

Illustration of the Dextral Hemicycle of Seven Tones

	Upper Quadrant. F Trichrome. F Trinomial.				Mutual Member.		Lower Quadrant. C Trichrome. C Trinomial.		
Dial Number	XII : I	: II	:		III	:	IV : V	: VI	
F Value	L : P	: T	:		TT	:	LD : PD	: TD	
C Value	TD : PD	: LD	:		TT	:	T : P	: L	
Tonic-Solfa	Fah : Doh	: Soh	:		Ray	:	Lah : Me	: Te	

These values are rational in the liminal equation of:—

LD as Tritensor	T^{III}
PD as Quadritensor	T^{IV}
TD as Quintensor	T^{V}

The mutual Bitensor III is the centre of the dextral hemicycle, and common to the super- and sub-quadrants.

It may be called the Yoke, and symbolised by the letter J (which stands for Jugator).

It symmetrically bisects the Dyads:—

$$(P : PD),\ (TD : L),\ (PD : P),\ (LD : T),\ \left({}^{F}_{C}DD : {}^{C}_{F}D\right)$$

Among the more obvious relationships are:—

$$F^{P}_{PD} = C^{PD}_{P},\quad {}^{F}_{C}\binom{T}{L} = {}^{C}_{F}\binom{LD}{TD}$$

The Hemicycle may be symbolised by H and its applement by πH.

J divides the Hemicycle into two Quadrants, the super (FQ) and the sub (CQ).

Their respective diametricals stand in inverted species relationship.

Regarding one quadrant as a Trichromal of first order (Quadrinomial of tones) it is evident that the three complementary tones in the other quadrant comprise the determinators of a monocyclic tertriadal matrix.

The order of J is neutral, but it may be conveniently regarded as "third" in the serial aspect.

The allocation of order settles the nomination of the Quadrant. The nominant quadrant may be regarded as consisting of an axial quadrinomial of tones (L : P : T : J) or as a trinomial of undetermined first-order intervals.

Its concomitant quadrant then supplies three determinators of specific relationship; when this relationship is reversed, the quadrant on the other side replaces the original concomitant, and the determinators are permuted; this quadrant is known as Orthoconcomitant, and the "Yoke" in this case is the Laxator (Ortho-yoke).

These two quadrants form, with the nominant, a Terquadrant group comprising all the available tones of the Tertriadal Matrix, and excluding the remaining Diametrical Quadrant to the Nominant.

The limitation of Q is the attainment of a second-order relationship, conflicting with the first-order grade of the cycle.

The limitation of H is the attainment of the neutral hemichrome W/2 by the extreme members L : TD, which, being a pseudo first-order chrome, conflicts with the grades.

The true first-order chromes about the hemichrome are given

by LL : L and TD : TTD, the tones thus introduced being found in the Quintriad, and situated in the laeval hemicycle.

It is seen that (in E.T.) the odd members of the F and C concomitant Tensors form a mutual Pentad.

Those of the Primes form a mutual Heptad, but the concomitant Laxators are only reconciled in a mutual " dodecanad "—an impossible chord.

The tonal " antipathy " thus evident between the Laxators may be compared with the dissonant phenomenon known as " False Relation of the Tritone " which is discussed in Successive Tonality.

The mutual Heptad is identical with the " white note " hemicycle, which is a particular form of the tertriadal matrix.

Since it forms the conventional basis of our system of notation and instrumental construction, it tends to appear a more *a priori* aggregation than the symmetrical or any other form. A more justifiable view is attained by abstract considerations.

The idea of concomitant reciprocity has been recognised for many years. It was pointed out particularly by Rameau, and discussed in detail by Von Oettingen, Hugo Riemann, and others.

Oettingen noted particularly the symmetry of species about " Ray " which he illustrated as a mirror reflecting chordal configurations; " Tonic " above and " Phonic " below, corresponding to the F and C Species.

The notion of coexisting species, differentiated into Predominant and Recessive, does not, as yet, seem to have been put forward. The idea, for instance, that the note " Doh " can be both " tonic " of C major and " mediant " of A minor at the same time would certainly appear ridiculous were it not for the principle of reversible Predominance and Recessivity, in which each aspect can be brought forward.

The rationality of the view regarding the Hemicyclic grouping as a Matrix, in distinction to other aspects, depends mainly upon the translatable direction.

This is properly considered in the chapter dealing with Progression, but it is an obvious experience that Pythagorean translation is a primary mode.

Each step around the cycle alters only one tone component, so that six components remain axial.

Beside the theoretical unity, there are practical advantages in being able to use a restricted number of tones for considerable extensions of key; and these advantages were all the more apparent in the days of unequal temperament.

The symmetrical conversity of the two concomitant quadrants may be shown in many ways, particularly by the dodecanal figure, and conveniently by the following groups:—

	Super-Dextral.				Yoke.		Sub-Dextral.		
Phase Number .	XII	: I	: II	::	III	::	IV	: V	: VI
Trinomials .	L	: P	: T	::	J	::	T	: P	: L
F Tertriad .	L	: P	: T	::	TT	::	LD	: PD	: TD
C Tertriad .	TD	: PD	: LD	::	TT	::	T	: P	: L
Chromes .	$-b^3$	: $-b^2$	: $-b^1$	::	b^0	::	b^1	: b^2	: b^3
	r^3	: r^2	: r^1	::	r^0	::	$-r^1$	: $-r^2$	: $-r^3$

By drawing the triads of the Tertriad (direct, and permuted) in a Pythagorean Dodecagon, the location of the Determinator apices is very evident.

Similar methods are applicable to the Antinominant Cycle.

The special relation of concomitance, presenting a Fundamental Scale of type LD=PD=FD and a Coincidental Scale of type TD=PD=CD, is called the Natural Scale.

It is a definitely chiral arrangement, which is usually taken as the nominantal basis of relationship.

The Fundamental Natural Scale is given by dial values:—

I, III, V, XII, II, IV, VI

which are read upwards in pitch, in successive first-order tones achromatically reduced to nearest position.

The corresponding Coincidental Scale is read downwards:—

V, III, I, VI, IV, II, XII

The " common species " scale will thus read from III.

The rationale of the Hemicyclic aspect is evident by its being a domain of restricted translatability, in contrast to the chordal aspect of the Tertriadal type.

The reference basis of translation, in staff notation, is that round the Pythagorean cycle, each step introducing only one actual note alteration into the matrix of seven tones.

When the two aspects are combined (as common in practice), it is observed that there is a natural preponderance or bias of

character towards the hemicyclic forms of Tertriadal type, both in scalar and chordal presentations.

The artificial appearance of the Symmetrical Tertriadal Matrix in notation emphasises the fact that this type is a somewhat late product of tonal evolution.

This suggests that the progress of future tonal developments tends towards an increasing independence of the species variability in the order partials, which may even lead to a revolution in the methods of notation.

The effect of Acoustical principles upon Tonality is a persistent tendency to conform the free variability of a continuous pitch flexion towards certain types; or rather, the tendency of certain types to survive.

SUBSECTION 5

THE SERIOPOLAR MATRIX

The tertriadal and Hemicyclic types of Matrix are seen to be founded upon the predominant characteristics of certain arbitrary groupings of elements, whereby the relationship of the members is definitely nominated within the system.

The bases of the Tertriadal aspect are:—

(1) Coherence of the section of the Series known as the Triad.
(2) Adherence of the Trinomial as a minimum portion of a cyclic relationship, which may owe its connectivity to either coherence (Pythagorean) or flexionic (Antinominant) adherence.

The basis of the Hemicyclic grouping is the approximation of a set of cyclically related tones (Pythagorean Hemicycle, or selections from the Antinominant cycle) to a special form of the Tertriadal type, the so-called "natural" scale of tones.

It is seen that in the Hemicyclic view, the prime triad, for instance, is given by a Zero-tensor, Tensor, and Quadri-tensor.

Both aspects are seen to reinforce and depend upon each other, having particular applications in use.

Thus, the Tertriadal aspect is that from which all varieties of chords are obtainable, while the Hemicyclic type represents an inter-matrical domain within which tones can move while conserved, and thus, by use of identical material, comply with the

great principle of economy, and maintenance of a stability of locus.

Also, the translation around the Pythagorean cycle is distinctively marked by the alteration of a matrical member at every step, upon which principle the signature of Staff notation is based.

Upon consideration, it may have struck the reader that both the Tertriadal and Hemicyclic aggregations are composite, involving at least two stages of relationship.

We now turn to an aspect in which the relationship of the members is unifold and wholly coherent, whose applicability depends upon an approximate coincidence with the other types.

This type of Matrix is conveniently known as the Seriopolar, for reasons which will be apparent.

The Series as a whole, forms a range of values descending in average audentity as the prime is departed from.

No definite basis of standard harmonic specification is as yet agreed upon, though statistical methods of treating psychophysical observations may provide a limited range of values.

In the actual case of the Harmonic Series, the intensity of the partials falls off rapidly, and the intervals converge more and more towards each other's parasyntonic tracts.

The members are to a certain extent evident, partials up to the ninth can be recognised by the trained ear, and by means of tuned resonators the series can be pursued further.

The average intensity of the members may be estimated on a physical basis as declining as the reciprocal second powers of the membership numbers, viz:—

$$1 : 1/4 : 1/9 : 1/16, \text{ etc.,}$$

but this is no measure of the auditory sense of loudness.

Investigations carried into the tympanum of the ear show that, in many cases, the intensity of the second partial exceeds that of the prime itself.

The principal characteristic of the series is its coherent unity. When the members are heard separately, it is noticed that the note-relationships present a quasi-matrical effect, a rough diatonic scale being possible in the fourth octave.

For convenience, we shall only speak of the odd series members, the even members being achromatic reproductions.

In the actual case of the Harmonic Series, the average intensity of the partials falls off rapidly, and the intervals converge more and more until the tones fall within each other's parasyntonic tracts.

Yet the members are individually perceptible to a certain extent (particularly in cases where some of them are absent, so that the gaps of pitch are wider). A particular case in mind is the "odd series" presented by linear vibrators having different terminators, uncompensated by "coning" (Clarinet, Stopped Organ pipes, etc.).

The characteristic property of Serial Coherence is retained to a certain extent even when the members are liminally mistuned.

Upon looking at the series, set out in notation, an approximation towards the Matrical form is noticed.

In particular, the nearest tones to the odd series of the Tensor, up to the thirteenth member, are given by the Hemicycle. Groups of this type present a quasi-matrical aggregation that is in itself coherent.

One of the triads in such a quasi-matrix is justly intoned, according to the selection of Prime. The further members, four in number, of the odd series, approximate somewhat to the other triad formations in the Tertriad.

These are the members, 7, 9, 11, and 13.

The ninth member is capable of factorisation, since it is the Tensor of the Tensor; *i.e.* the Bitensor of the Prime. This is the relationship between the Polar tones of the Trinomial, since $L_9 = T$.

The nearest tone in E.T. to T_7 is L.

The relationship thereby established was particularly pointed out by Dr. Hugo Riemann, who realised its limiting capabilities. The formula $T = L_9$, $L = T_7$ may, therefore, be known as "Riemann's Equation."

It may be noted that $9 \times 7 = 63 = 2^6 - 1$.

The principle of Seriopolar aggregation depends upon the fact that the Series of each pole contains the opposite polar tone within the first seven odd members.

In respect to the tertriad, the effect is to bind the Polar elements in a coherent relationship independently of the mutual alliance with the Centron.

It is observed that the members:—

$$\begin{array}{lcl} T(7 : 9 : 11) & \text{approximate to} & L(1 : 5 : 3) \\ L(9 : 11 : 13) & ,, & T(1 : 5 : 3) \end{array}$$

so that the serial members of one pole present a "weak" approximation to the opposite polar triad.

The frequency relations may be compared.

7 : 9 : 11	equals	252 : 324 : 396
	Difference	0 : 9 : 18
1 : 5 : 3	equals	252 : 315 : 378
	Difference	0 : 7 : 14
9 : 11 : 3	equals	252 : 308 : 364

from which it is perceived that a mean of the polar values approximates closely to a Just Triad.

An important difference between the polar approximators should be noted.

T_7=Tᗡ is the first new series member, in direct relationship, beyond the triad; the number is "prime."

L_9=LTT is a "further" series member, transcends the octave, and is, moreover, a secondary "first-order" relationship, since 9 factorises.

The Prime (Laxator Tensor) intervenes in the latter relationship, which is directed *via* the Centron, and not, as in the case of the Contra-determinator, by a predominant independent line of connection.

Further approximate equations may be noted:—

L(3 : 7 : 9) to P : ◗ : T
T(11 : 13 : 16) to P : Đ : T

In these cases, the permuted and indeterminate forms of Centron Triad are presented by the polar serials, so that the three triads of the tertriadal matrix are approximated by both polar series up to the 13th member.

It should be noted that the partials of a Series are not heard separately except under special circumstances, and the evident mistuning of the approximations is distinctly apparent.

The Tertriad is thus suggested:—

	L Polar.		Centron.		T Polar.
	L : LD : P	:	PD	:	T : TD : TT
Series of T	7 : 9 : 11	:	13	:	1 : 5 : 3
Series of L	1 : 5 : 3	:	7	:	9 : 11 : 13

By expression of the exact frequencies in Binary Logarithms, comparison may be easily effected. This can be done on a Slide rule, or squared paper, and the results shown graphically.

The general appearance of approximate triads in the upper serial members is to be expected from the disposition of the convergent values in pitch.

The effect is that of a nominant triad accompanied by a fading train of pseudo-triads; of predominant, indeterminate, and recessive species.

The essential basis of a Seriopolar aspect is the fact that the polar series presents a strong polar triad, a quasi-triad of opposite polarity, and a still weaker quasi-centron, plus a weaker train of further forms.

Quasi-recurrence and axiality can be seen in the following table, in which series members up to the 15th are taken together with their own series up to the same value.

The names of the tones are given in their nearest Solfa representatives, and the products of the frequency numbers read diagonally downwards from left to right.

P	4	5	6	7	9	11	13	15
	Doh							
4	16							
	Me	Me						
5	20	20						
	Soh	Se	Soh					
6	24	25	24					
	Ta	Te	Te	Ta				
7	28	30	30	28				
	Ray	Ray	Ray	Ray	Ray			
9	36	35	36	35	36			
	Fae	Fe	Fah	Fah	Fe	Fae		
11	44	45	42	42	45	44		
	Laah	Lae	Lah	Soe	Lah	Lae	Laah	
13	52	55	54	49	54	55	52	
	Te	Doe	Doe	Doh	Doh	Doe	Doe	Te
15	60	65	66	63	63	66	65	60
		Re	Mae	Ma	Me	Ma	Mae	Re
		75	78	77	81	77	78	75
			Fe	Fe	Soe	Soe	Soh	Fe
			90	91	99	99	91	90
				Lah	Tae	Te	Tae	Lah
				105	117	121	117	105
					De	Ray	Raay	De
					135	143	143	135
						Mee	Me	Mee
						165	169	165
							Soe	Soe
							195	195
								Le
								225

The General Series may be divided up into Order groups:—

Zero	1 : 2
First	3 : 4
Second	5 : 6 : 7 : 8
Third	9 : 10 : 11 : 12 : 13 : 14 : 15 : 16

This enables an estimate of determinant audentity to be made.

The series members 7 : 11 : and 13 are all considerably " out of tune " in respect to the other members.

P_7 is the Contra-determinator; it is situate between LD and LL, the approximation to LL being nearest, as seen by the respective frequencies 60 : 63 : 64.

It is roughly approximate to the diametrical prime indeterminator πPD, and it bisects R, being the harmonic mean of 6 : 8 or T and WP.

P_{11} is situate between L and TTD, the ratios being 128 : 132 : 135. It thus approximates to the Bitensor indeterminator TTÐ, and it harmonically bisects the V chrome of (PD : T).

P_{13} is situate between L◗ and LD, the ratios being 192 : 195 : 200. It thus approximates to the Laxator indeterminator LÞ, and it harmonically bisects the " hypo-chrome " of T : Pᗡ.

P_9 bisects the G chrome (P : D) into two oscillants differing by a Comma.

P_{15} bisects the Super-oscillant Pᗡ : WP.

The nearest approximations are given by the Tensor series, which, up to the thirteenth member, falls somewhat near the Hemicyclic aspect, in which the prime of one species is identical with the determinator of the other, the yokes being common to both.

Tensor Members	1 :	2 :	5 :	7 :	9 :	11 :	13
F on Dial	II	III	VI	(XII)	IV	(I)	(V)
C ,,	IV	III	XII	(V)	II	(V)	(I)

These members are all on the right-hand half of the dial; the members within the brackets are considerably mistuned.

The next nearest approximation is given by the Laxator series up to the 15th member:—

Laxator Members	1 :	3 :	5 :	7 :	9 :	11 :	13 :	15
F on Dial	XII	I	IV	(X)	II	XI	III	V
C ,,	VI	V	IX	(VIII)	IV	VII	III	I

which approximates, as far as the thirteenth member, to a permuted tertriad, oppositely as to polars.

The mistuning is somewhat less evident with E.T. values.

The Series of P, and of the further polar nomials TT, LL, etc., approximate to a lesser extent to the matrical types arrived at by other processes; so that, although the actual series are just as coherent, the matrical value is less evident and need not therefore be particularly considered.

The series of the two poles present individually the Serial, and conjointly, the Seriopolar, Matrix.

The tones P_{11} and P_{13} have not as yet received any distinctive names. Their chromo-schismic function is evident, but it is convenient to regard them as the representatives of TTÐ and LÐ respectively.

It is to be noted that the Quasi – B interval given by P(9 : 13) is nearly equal to the second-order chrome M, but its harmonical bisection by P_{11} strengthens its appearance as a first-order chrome of a quasi-triad.

In the Coincidental Species, the actual coherence is not evident much beyond C_7.

This leads to the particular view known as the Seriopolar Aspect, in which the terms of each pole are neglected beyond the seventh, and the Matrix is regarded as made up of the Polar Tetrads T(1 : 3 : 5 : 7)+L(1 : 3 : 5 : 7), T_7 being regarded as the Laxator.

The Tertriadal Axes are given by T_1 and L_3, and the Centron Determinator does not appear, although weakly presented in permuted form by L_7.

The essential point is that the Seriopolar aspect corresponds to the Tertriadal in maintaining the polar distinction, but that the audentity of the elements Core and Envelope is reversed, while the connection of the polar triads by means of the Riemann equation is emphasised, *i.e.* T_7 sounds sufficiently like L to represent it.

The actual formulation is due to the predominance of the audentity (T : TƆ) over that of (L : LTT), partially attributable to the average intensity in the harmonic series, but much more owing to the fact that the Contra-determinator is an entity, *i.e.* that 7 is a prime number, while 9 is capable of factorisation into 3×3, so that L_9 sounds like the tensor of its centron triad.

In the opposite relationship:—

L(1 : 5 : 3 : 7 : 9) + T(1 : 5 : 3 : 7 : 9), where $T = L_9$, $(L_7 : T_3)$ is achromatically a relationship of 28 : 27. This is an approximation upon the boundary of liminality: actually highly discordant.

The Vertication and identity of the components is evidenced by the effect of the interval (L : LⱭ) as part of the fundamental "Dominant Minor Thirteenth." In this case it is a well-known fact that the laxator sounds best when below its contra-determinator in pitch.

If L be regarded as TⱭ, then LⱭ=TⱭⱭ, the Tensor Bicontra-determinator.

As a series tone T_{49} is, of course, not audible, but some points may be noticed.

It is nearly the harmonic indeterminator of the Prime Triad, since:—

P : TⱭⱭ : T :: 40 : 49 : 60

P◗ : TⱭⱭ : PD :: 48 : 49 : 50

The frequencies of the complete seriopolar "heptad" are given by:—

T(P :	D :	T :	Ɑ(P :	D :	T :	Ɑ)
16 :	20 :	24 :	28 :	35 :	42 :	49

A quasi-octave is formed between TT=24 and TⱭⱭ=49. This is a connective element of some importance.

The rationality of the seriopolar aspect is perhaps not so easy to comprehend as that of the other aspects.

The peculiar inversion of the tertriadal idea will be evident when the general constitution of the matrix is discussed.

A point of some interest is the identity of the laxator contra-determinator with the so-called dominant minor thirteenth, which appears in Staff notation (usually) as an "augmented fifth."

The old controversy as to the harmonic relationship and proper notation of the note (see Dr. Alfred Day's Treatise and various other works) is beside the mark if one regards the disputable tone as neither prime tensor nor determinator, but simply as the contra-determinator of second order.

Further evidence on this point must necessarily be deferred until progression is considered.

If the Laxator be considered in its aspect of Reciprocal Tensor (T^{-1}), another view of Seriopolar aggregation is evident.

It is noted that P approximates to T_{11} and T consequently to P_{-11}, so that the binaxes of the Tertriad "reappear" upon serial extension.

Writing the Centron B and taking converse series, we obtain:—

$$F\begin{cases} P \\ TTT \\ T\text{ᗡ} \\ TT \\ TD \\ T \end{cases}$$

$$C\begin{cases} P & \\ D & = FL\text{◗} \\ T & = FL \\ \text{ᗡ} & = FTT \\ TT & = FT\text{◗} = F\text{ᗡ} \\ L & = FT \end{cases}$$

In the converse arrangement, we have:—

Fundamental Series of FT (odd members)—

Soh : Te : Ray : Fah : Lah : Doh

Coincidental Series of FP—

Doh : La : Fah : Ray : Ta : Soh

The two forms of Envelope Tetrad thus presented, may be compared with those obtained by the Tertriadal method of aggregation; and the inherence of the polar triads in mutual Hexad is evident.

The Axis of the grouping is the Dyad (Ray : Fah), which is symmetrical about the tone "Mae," the Prime Indeterminator, *i.e.* the centre of Tertriadal symmetry.

When, therefore, we speak of polar triads as conserved in a matrix, we imply this connection by means of the Link Dyad (Ray : Fah), as well as by way of the Centron.

This Link Dyad appears in the two prominent radical polar tetrads—

(T : TD : TT) + L

(L : LD : LT) + TT

which are familiar to harmonists (in fundamental case) as the chords of the Dominant Seventh and Added Sixth, respectively.

The Tensor Tetrad	T	: TD	: TT	: Tᗡ
Fundamental	Soh	: Te	: Ray	: Fah
Coincidental	Lah	: Fah	: Ray	: Te

The Laxator Tetrad appears as a Commute Radical Tetrad on the Prime (since $P=T^0$).

$(P:D:T:D^{-1})$	P^{-1}	$:D^{-1}$	$:T^{-1}$	: ᗡ$^{-1}$
Fundamental	Doh	: La	: Fah	: Ray
Coincidental	Me	: Se	: Te	: Ray

The enantio-centric functions of the Link Dyad, in relation to a cyclical conservance of the Matrix, will be discussed in detail later. The principal point now to be noted is the ground of rationality in the Seriopolar aspect.

SUBSECTION 6

THE GENERAL MATRIX

The General Matrix presents the common abstract characteristics of its component types, appearing as a limited (seven-toned) aggregation of tones referable to a nominant locus.

The locus itself is referable to a centre of symmetry, and the essence of conservation is the fact that the component chords, intervals, and tones retain their nominance when separated from the aggregate.

The principle of limitation is the cyclic formation of the aggregate, with the consequence that every relationship is dual, *i.e.* any pair of inherent tones can be connected by at least two independent paths.

The basis of limitation is due to the E.T. equation of chromal orders.

The tertriadal basis is the trinomial of tones in first-order relationship, or of first-order chromes, which is similar to the hemicyclic connection.

Each tone is connected to its neighbours by inherence in a first-order chromal dyad.

But there is a secondary connection in the minimal group of seven tones, afforded by the approximation to second-order relationship.

This, in the Quadrant L : P : T : TT, is evident between the terminal members L and TT.

The Hemicycle presents a pair of quadrants, the tones of one determining the chromes of the other.

The Seriopolar aspect shows how the secondary connections L : TT and their analogues may occur in the Series of the Poles and in the converse Series of the Axials, P and T.

The mutual dependence may be expressed in various ways.

Type of Matrix.	Primary Connection.	Secondary Connection.
Tertriadal	(L : P :: P : T :: T : TT)	(TT : L=V or Y)
Hemicyclic	(Sub to Super Q via J)	(Mutual Determinance)
Seriopolar	(Series Coherence)	($L=T_7$, $T=L_9$)

The semi-matrix or quadrinomial of four tones is itself cyclical, but its complement the trinomial is not. (L : T = an oscillant, *i.e.* a relationship not of chromal degree.)

This implies the necessity of triadal extension in order to complete the minimum conserved aggregate.

The general economical grouping of univalent members comprises three triads, two quadrinomials, and two radical or converse tetrads.

Three triads present nine tones, of which two are axial (P = LT) (T = PT).

Two quadrinomials present eight tones, of which one is axial, J = FTT = CTT.

Two radical tetrads present eight tones, of which one is axial, $L = T_7$.

Two converse axial tetrads present eight tones, of which two are axial, $CP_3 = FT_7$, and *vice versa*.

Upon continuation of either series to the thirteenth member, we obtain the Prime Determinator, thus completing the seven tones of the General Matrix in all aspects.

These seven tones, with their seven first-order chromes (six serial and one pseudo in the H aspect), constitute an economical cyclic group, out of which any selected configuration whose inherent nominance is recognised may be regarded as conserved in the particular matrix.

The permutable variability of the determinators allows a number of alternative forms, but a determinator cannot be

simultaneously permuted and unpermuted without creating the peculiar effect known as "static false relation."

The possible group of ten members is therefore restricted to the actual seven.

If, by any means of extension, the further members LL and TTD are introduced, a further "second order" linkage is established by the interval in question equating to G or M.

In this case the univalence of matrical members becomes indiscriminate in the twelve tones of the E.T. Domain.

A clear distinction is maintained between the Septomial and any further matrical aggregation, such as the Nonomial.

Nevertheless, the Tri-heptad presents such a case of an extended matrix, in which secondary polar triads and tones are still recognised as conserved.

The possibility of using the Nonomial scale in contrary motion with the Septomial enables the nonomial terms to be recognised as conserved.

Example:—

Doh : Te : Ta : Lah : Soh	Soh : Lah : Te : Doh
Doh : Ray : Me : Fah : Soh	Soh : Fe : Fah : Me

The Domain of twelve tones appears in the determinator permutance of the Hemicycle.

A Hemicycle consists of the Nominant and Concomitant Quadrants. The Orthoconcomitant quadrant presents the permuted determinators of the nominant quadrant.

Thus we have:—

	F	C
Nominant Quadrant . . .	Fah : Doh : Soh : Ray	Te : Me : Lah : Ray
Concomitant (Determinators) .	Lah : Me : Te	Soh : Doh : Fah
Orthoconcomitant	La : Ma : Ta	Se : De : Fe

This gives thirteen tones, reducible by the identity of πJ (Se = La) to the twelve.

To recapitulate, a Tertriad consists of a Centron Triad and its two adjacent Pythagorean polars.

A Quintriad similarly embraces the two next Pythagorean Triads.

A Tri-heptad consists of three tertriads, of which the centrons are respectively the three triads of the original tertriad. It thus

actually consists of five triads, and is identical in form with the Quintriad.

The principle of limitation is the restriction of a group within the smallest cycle of relationship.

In the case of the Quadrinomial of tones, this is given by:—

$$L : TT = V \text{ or } Y$$

The Hemicycle contains only one semi-octave or pseudo-first-order relationship, which excludes first-order chromes containing either of these tones, thus limiting the triads to the six (three of each species) in the Matrix.

This introduces the general canon of limitation, viz. that aggregates presenting semi-octaves cannot contain the first-order chromes (and therefore the triads) involving either of the two notes in the dyad.

On this criterion, a "Septriad" or Ter-tri-heptad aggregate thus precludes the Centron Triad, hence cannot be recognised as a Matrix.

The Quintriad and Tri-heptad have a restricted matrical applicability, although the criterion precludes polar triads. There are, however, two progressional directions, so that bivalence can be made clear, and the entities of perception discriminated, as in the particular case of contrary motion in melodic minor scales.

The limitation principle of the Matrix is expressible in the phase criticals of the domain cycle.

The Hemicycle appears as a group determinately excluding the laeval complement, and as divisible into two quadrants of opposite species, or different order.

The principle of phase imitation expressed in terms of the "tangent" criterion (as discussed in Chapter VII.) is therefore no imaginative fiction, but a real indication of the characteristic limits of aggregation.

Thus, at the Quarter cycle, the tangent direction is orthogonal, and represents independence of the grade connection (first order) by the introduction of a second-order direct relationship.

At the Hemicycle (as given by the diametrical "horns" L and TD) the tangents are parallel, but oppositely directed in chirality, representing an entire reversal of the grade connection by the attainment of a pseudo-first-order direct relationship.

The limitations of the Seriopolar type have already been discussed; the rationale of this type is not evident without consideration of others, being formulated by the approximation to the general type.

It may be noted that P_{13}, as limiting member, is separated from $P_{16}(W_4)$ by two factorisable members, $14 = 2 \times 7$ and $15 = 3 \times 5$.

As illustrative of an aspect in which the Hemicyclic and Seriopolar types are concerned, it is to be noted that $(FT_{15} : CT_{15}) = G$ or M.

Also that FT(5 : 9)=C(L : T) and *vice versa.*

The general Matrix involves the liminal mistuning of many of its component tones.

Since the E.T. mistunes all the components except the Prime, it tends to smooth out the individual librations of each typical aspect.

General Table

E.T. Values—

Fundamental .	Doh : $\frac{\text{De}}{\text{Ra}}$: Ray : $\frac{\text{Re}}{\text{Ma}}$: Me : Fah : $\frac{\text{Fe}}{\text{Sa}}$: Soh : $\frac{\text{Se}}{\text{La}}$: Lah : $\frac{\text{Le}}{\text{Ta}}$: Te
Coincidental .	Me : $\frac{\text{Ma}}{\text{Re}}$: Ray : $\frac{\text{Ra}}{\text{De}}$: Doh : Te : $\frac{\text{Ta}}{\text{Le}}$: Lah : $\frac{\text{La}}{\text{Se}}$: Soh : $\frac{\text{Sa}}{\text{Fe}}$: Fah
Tertriadal Aspect .	P : πT : TT : P◗ : PD : L : πP : T : L◗ : LD : T◗ : TD
Quintriadal Aspect	P : LL◗ : $\frac{\text{TT}}{\text{LLD}}$: P◗ : PD : L : TTD : T : L◗ : $\frac{\text{LD}}{\text{TTT}}$: $\frac{\text{T◗}}{\text{LL}}$: TD

Hemicyclic Aspect (Arbitrarily Fundamental) .	J^{-2}	—	J^0	—	J^2	J^{-3}	—	J^{-1}	—	J^1	—	J^3
Dial Numbers .	I	VIII	III	X	V	XII	VII	II	X	IV	XI	VI
Tensor Series .	(11)	—	3	—	(13)	(7)	—	1	—	9	—	5
Laxator Series .	3	—	(13)	(7)	—	1	—	9	—	5	—	(11)
Seriopolar . .	21	—	3	49	—	(7)	—	1	—	35	—	5
Converse Axial .	1^{-1}	—	$\frac{3}{7^{-1}}$	—	—	$\frac{7}{3^{-1}}$	—	1	5^{-1}	—	—	5

The frequencies of the above terms can be calculated, and the pitches determined from a table of logarithms, or directly from the compendious tables of A. J. Ellis, Carl Stumpf, or other authorities.

By means of a Slide-rule the values can be graphically plotted side by side, when the differences of tuning are directly perceptible.

CHAPTER X

SUBSECTION 1

CORE AND ENVELOPE

THE Matrix has been defined as a particular group of seven related tones (Septomial) exhibiting particular properties, and appearing in three characteristic types of inter-relationship.

Within the matrix " conserved " operations are possible and the matrix itself can move in a domain which is infinitely extended in just intonation, but which is reduced to the Dodecanal by E.T.

The locus of a matrix is nominated by a particular component tone, which may be regarded as somewhat analogous to the mechanical centroid of a mass.

We may now proceed to examine the internal structure of a general matrix and its equilibrative distribution about the centroid.

This examination is directed not so much towards the typical components, whose syntheses have been already discussed, as to a general analysis based upon the comparative functions of a matrix and its parts.

A complete grasp of the subject cannot, perhaps, be attained until the progressional considerations are taken into account, but some of the preliminary principles may be surveyed.

The tertriadal aspect introduces us to a trinomial consisting of the nominating " centroid," viz. the Centron (Prime Triad) and its " excluded " companions, the dextral and laeval polar triads.

These latter, being of similar character but opposed in sign, may be considered to constitute a non-centron group of components.

The Yoke and non-Yoke members of a Hemicyclic matrix may similarly be regarded.

In the Seriopolar aspect we are introduced to a binding element between the polar components, the centron components appearing as relatively excluded thereby.

It therefore appears that the Matrix may be generally regarded as made up of two contrasting complementary portions, viz. the Centron triad and the summed polars.

These portions are united by the axial bi-valent members P and T; the polars themselves being united by various bonds, principal among which is the serial statement of Riemann's equation in the form $T_7 = L$, $L_9 = T$.

Another aspect in which the polar components are united is particularly evident in the symmetrical type of matrix, where the series of opposite species are projected in converse directions from T and P.

The coincidental triad of a given prime appears as the permuted fundamental laxator triad, and *vice versa*.

Taking the Laxator triad in this form, and extending it to the Tetrad, we have (reading out from the Prime):—

F Species	Doh : La : Fah : Ray
C Species	Me : Se : Te : Ray

The Tensor Radical Tetrad (reading in converse direction):—

F Species	Soh : Te : Ray : Fah
C Species	Lah : Fah : Ray : Te

The common Dyads, F (Fah : Ray), C (Te : Ray), are to be noted, also the quadri-valence of Ray.

In the above case, Riemann's equation reads:—

$$FP_7 = CP_9 \text{ and } \textit{vice versa.}$$

The Heptad may be written in two ways, showing the dual connection.

$$(1)\ L : L^{D}_{\text{◗}} : P :: P : P^{D}_{\text{◗}} : T :: T : T^{D}_{\text{◗}} : TT$$

$$(2)\ T : TD : TT : T^{\text{ᗡ}}_{L} : \frac{TTT}{LD} : P : L\text{ᗡ} : T$$

$$T \;.\;.\;.\;.\;.\; P^{D}_{\text{◗}} \;.\;.\;.\;.\;.\;.\; P$$

In both these forms, the resolution of the Septomial aggregate into two contrasted groups, connected by the Axes P and T, is evident.

The second of these two aggregations may be written in the

reversed aspect of Riemann's equation, when the "Pentads" of $\frac{L}{T}(1:3:5:7:9)$ present the "pseudo" permuted Centron triad.

$$\text{L : LD : P : Lᗡ : T :: T : TD : TT : L : LD}$$

$$\text{P} : \frac{\text{Lᗡ}}{\text{P◗}} : \text{T}$$

To distinguish this analytical view of the Matrix from any other aspect, the Centron Triad may be termed the CORE, and the excluded terms, its ENVELOPE; the two being united by the Axes Prime and Tensor.

In a system of symbolisation to be explained later the Envelope may be denoted by the letter E, while its Core may be looked upon as a Zero Envelope, and written E^0.

These names are suggested by the grouping when the Matrix is written out in Scale position, *i.e.* achromatically reduced to nearest pitch position.

The three tones of the "Core" are enveloped by the four tones constituting an excluded Tetrad, two components being contributed from either pole.

When the two axials are added to the Tetrad, the complete Envelope consists of the two Polar triads, forming a Hexad, the Core determinator being the only excluded member of the Matrix.

Core . . $\text{P} : \text{P}^{\text{D}}_{\text{◗}} : \text{T}$

Envelope. . $\text{T}^{\text{D}}_{\text{◗}} : \text{TT} : \text{L} : \text{L}^{\text{D}}_{\text{◗}}$

It is to be noted that the members of Core and Envelope interpenetrate, and thus act as insulators separating the components of each other.

In order to symbolise *Envelope* Tetrads and Hexads, the triangular representation of triads may be extended to the Tetragon and Hexagon, standing upon an angle.

Radical chords, tetrads, pentads, hexads, and heptads, can be distinguished by polygons having the same number of sides, but standing upon a base.

In both cases, the nominant of the enveloped Core, or the Radical Prime, can be inscribed within the polygon.

The chordal system so far considered appears as a theoretical grouping determined by given conditions. The configurations,

therefore, present a somewhat artificial aspect, inviting criticism.

It is now advisable to apply the actual criteria of experience, and to view the real contrasting characteristics of the Core and Envelope.

This necessitates reference to the problem of concordance versus discordance, as stated in the form Serial versus Homochromal.

In the abstract view, terms of zero order do not appear: *i.e.* achromatics are regarded as isotonal.

The Serial Chromes of first order are B and R.

The corresponding " homochrome " is W/2, which, as a semi-octave, is contrasted with W, the " period."

Being the mean of B and R it is neutral in respect to Species and Polarity, and is thus pluri-valent.

This interval is given by the E.T. Dyads:—

(D : ᗡ) (◗ : ◖)
(L◗ : J) (P◗ : L) (T◗ : PD)

The Serial Chromes of second order are G and V, their applements being M and Y.

The second-order " homochrome " B/2 (and similarly R/2) does not appear in the E.T. system.

The E.T. Domain restricts consideration to the contrast between the two ways of bisecting an Octave, viz. the Harmonic or Serial, and the Arithmetical or Homochromal.

These two opposed elementary intervals appear as discriminative agents between the Core and Envelope.

The concordant coherence of the Core is evident; it stands for the greatest number of independently perceived serial concordances.

The Envelope Tetrad coheres in virtue of its approximate appearance in either polar series, but its elements contain the following intervals.

Taking the Symmetrical Form as basis, we have:—

	TD	: TT	: T^{-1}	: $T^{-1}D^{-1}$
i.e.	TD	: TT	: L	: L◗
F Species	Te	: Ray	: Fah	: La
C Species	Fah	: Ray	: Te	: Se

It is seen that the intervals are multiples of the minimum concordant chrome V, the chord itself being a homochrome

which may be regarded as a Triple Violet or Triple Yellow, or, by bundle analysis, as a sum of Y, W/2, and V.

Upon achromatically repeating one member, we obtain a Quadruple V or Y, which extends an octave, forming a tetragonal cycle denoted by a square on the Dial in either Pythagorean or Antinominant terms.

In consequence of its characteristics, the Enveloping Tetrad may be considered as a chord almost equalling the Triad in importance.

Relatively to the Core, it exhibits the opposite extreme of chordance effect while maintaining parity of euphony.

The old criterion of con- versus dis-cordance, on the basis of agreeability, is seen to be quite beside the mark.

The Enveloping Tetrad contains two permutable determinators, and can thus appear in four different forms:—

(1) Symmetrical (Homochrome) .	T(D : T) : L(P : ◗)
(2) Hemicyclic (Concomitant Pentad)	T(D : T) : L(P : D)
Dial values	(VI : III : XII : IV)
(3) Anti-symmetrical . . .	T(◗ : T) : L(P : D)
(4) Totally permuted Hemicyclic .	T(◗ : T) : L(P : ◗)

Each of these forms exhibits peculiar characteristics.

(1) The Symmetrical Tetrad has already been discussed. In consequence of the homochromality of its intervals it is seen to be quadri-valent in four matrical loci, and is therefore an important (and somewhat overworked) chord in translation.

There are three independent arrangements possible with twelve notes of the Dodecanal.

The Symmetrical Tetrad may be regarded as an equation in Species, composed of C(PD+PT)+F(TD+TT).

Again, by regarding the chord as composed of two hemichromes W/2, we have C(PD+ᗡ)+F(TD+Tᗡ). These two orthogonal aspects of the chord are seen to be inseparable from the association as the sums of three chromes (V, Y), and the sum of two hemichromes (W/2).

(2) The Hemicyclic form is seen to consist of "natural" notes (F; Te : Ray : Fah : Lah; C; Fah : Ray : Te : Soh).

The two concomitants together present the mutual or "axial" radical pentad.

(3) The Anti-symmetrical form is the most discordant in effect. It is of relative unimportance, sounding, under ordinary circumstances, like the upper part of a Tensor heptad in the Laxator key.

(4) The totally-permuted form is identical with a Radical Tetrad whose locus is turned through a phase quadrant, the operation being indicated by (XI/II) or (LL/T). The importance of this aspect will be discussed under the heading of Ortho-Concomitance.

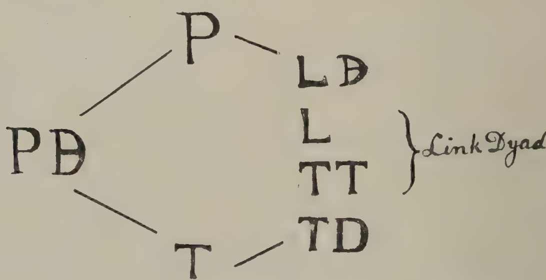
The relationships between the Core and Envelope may now be briefly discussed.

Among some of the most obvious points, the following may be noted:—

In passing along the Septomial Scale between the limits TD and LD, there is an alternation between Envelope and Core, thus undulating between association with concordance and discordance.

The true centroid of symmetry is the Prime Indeterminator. Since, however, this tone is not found in the E.T. it is represented by the two determinator values. In practice the Coherent basis of the Core, the Prime, is the nominator of locus.

The Matrix may be written so as to exhibit its dual connection in a cyclic form.



This exhibits the bonds between the polar triads, both by way of the Centron, and by Seriopolar Coherence.

The dextral and Laeval elements of such a cycle are thus graphically contrasted.

It will be noted that the cycle is the simplest completely recurrent harmonic formula in use. In the Fundamental form

(Major Key) the sequence of Tonic : Subdominant : Added Sixth : Dominant Seventh : Tonic is well known.

The reason for writing the scale between the polar determinators in their further position, instead of tonic to tonic, will be discussed later.

We may here observe that the conjunct fluent formed between these second-order tones is of a neutral species.

It is capable of being either A, O, or a super-oscillant equal to V, according to the permutable variability of the components.

The varieties of scale possible with two core, and four envelope, permutabilities, are eight in number, represented generally by the symbolic expression:—

$$\mathrm{T}^{\mathrm{D}}_{\text{◗}} : \mathrm{P} : \mathrm{TT} : \mathrm{P}^{\mathrm{D}}_{\text{◗}} : \mathrm{L} : \mathrm{T} : \mathrm{L}^{\mathrm{D}}_{\text{◗}}$$

Amongst these eight forms, four in particular are easily recognised, viz.:—

(1) TD : P : TT : PD : L : T : LD

The "Natural" Scale, in which the concomitant F and C forms correspond to the Major and Relative Minor (Æolian or Declinear-Melodic variety) Modes.

$$(2)\ \mathrm{TD} : \mathrm{P} : \mathrm{TT} : \mathrm{P}^{\mathrm{D}}_{\text{◗}} : \mathrm{L} : \mathrm{T} : \mathrm{L}\text{◗}$$

The Symmetric form, the scale corresponding to the Tonic Minor (Harmonic Form) Mode, and its "Picardian" or "Hungarian" variant.

(3) F(TD : P : TT : P◗ : L : T : LD)
C(T◗ : P : TT : PD : L : T : L◗)

Core and Envelope opposed, presenting the Aclinear Minor Mode (Tonic and Relative forms, respectively).

(4) T◗ : P : TT : P◗ : L : T : L◗

The totally permuted form, corresponding to the Declinear Tonic Minor Mode.

These forms are recognised as the basic scales of practice, although other forms appear, as transitory variants, during the course of musical work.

The relationship between scalar direction and determinator permutation is important, and is discussed later.

Enough has been said to show that the idea of a Matrix composed of Core and Envelope is not a mere theoretical fiction, but corresponds to a real and enlightening expression of relationship.

The notion of a Matrix may take various forms according to the aspect in which it is viewed.

The general idea is that of a group out of which selected configurations can be extracted.

These selections are nominated by the locus of the Matrix, which is represented by a component member acting as a centroid, and itself nominated both with respect to the Domain in which it can vary, and to its absolute pitch position.

This last is, of course, only expressible relative to some external standard, such as the auditory, useful, or any arbitrary limits of the range, and by implication, to the unit of time in which frequency is expressed.

An interesting illustration of the idea of matrical distribution about its centroid is afforded by mechanics.

A given mass may be equatively distributed about its mechanical centroid in two ways, *i.e.* the latter may be situate within the mass, as in a solid sphere, or enveloped as with a spherical shell.

This illustrates the way in which both the Centron and its Envelope are equatively distributed about the matrical locus.

A centroid behaves to acceleration, etc., as the mass, and mechanically represents it; *e.g.* the attraction of a weight by the earth is along the line joining their respective centroids.

When the centroid of a body is unaffected by the position of the mass, the body is said to be stable. If it is found that at a certain line of position the mass becomes unstable, this line traces out the natural limit of the system.

The importance of this illustration in showing the relationship between the inviolability of the Core, and the limits of conserved chordal extension, will be considered in a subsequent section.

Matrical extension on the Pythagorean basis is by no means limited to 3, 7, or even 12 independent elements, although these groupings coincide with certain experiential conditions of considerable audental predominance.

A matrix may be somewhat fancifully compared to a solid core of trinomial elements, surrounded by a liquid sea of septanomials, and enveloped in an atmosphere of dodecanomials: the whole grouping having a definite locus in the "space" of its Domain.

The general idea of a Tetrad Envelope, as viewed from a Core Triad, is that of a chord whose four tones are situate respectively somewhere in the four pitch spaces outside and between the three triad tones.

Whatever these "floating" tones may be, they are distinctly recognised as non-centron.

Such spaces may be named and symbolised, irrespective of the species of the Centron triad, as follows:—

E/– Super-core (super-tensor or super-prime)
E/D Super-determinator
D/E Sub-determinator
–/E Sub-core (sub-prime or sub-tensor)

In tonality, as at present employed in musical manifestation, we are limited to the twelve E.T. tones for choice of tetrad components.

The available notes may be given in Tonic-Solfa:—

	F		C	
	Educt.	Adduct.	Educt.	Adduct.
Super-core .	Lah La	Lah La	Fe Fah	Fe Fah
Super-determinator .	Fe Fah	Fe Fah Faa	Re Ray Ra	Re Ray
Sub-determinator .	Re Ray Ra	Ray Ra	Te Ta	Tee Te Ta
Sub-core .	Te Ta	Te Ta	Se Soh	Se Soh

Thirty-two forms of Tetrad chord are thus possible.

The selective agency is determinate tonality.

From the Pythagorean aspect, the Envelopes of First Grade are those tones comprised in the Polar Tetrads, less the Axes.

These tones form the discord of the normal "Perfect or Terminal Cadence."

The Envelopes of Second Grade comprise also all the tones of the Tri-heptad or Quintriad Matrix, introducing the additional tones of the Bitensor Determinator, Bilaxator, and Permuted Bilaxator Determinator.

The Antinominantal Polars provide further forms of Envelope.

In considering the envelopes of radical form, it should be noted that FL◖ =CPᗡ in just intonation.

To this fact we largely owe the predominance (in F species) of the " Chord of the Added Sixth " on the Laxator.

Envelopes of Second Grade, and Antinominantal Envelopes are not usually employed in terminal cadences, but if the latent intermediate step be sufficiently evident there is no reason why they should not be so used.

For many purposes it is possible to exchange, or replace, one form of Tetrad for another, both in extension and inversion.

The Symmetrical Tetrad is iso-homochromal in any inversion, and there are three such chords in E.T., viz. the Symmetricals about the Centron and Polar Triads ⟨L⟩ ⟨P⟩ ⟨T⟩

A familiar case of Tetradial Exchangeability may be noted. Fundamental example:—

$$\left(\begin{matrix}\text{Fe}\\ \text{Fah}\end{matrix} : \begin{matrix}\text{Re}\\ \text{Ray}\end{matrix} : \text{Doh} : \begin{matrix}\text{Lah}\\ \text{La}\end{matrix}\right)$$

Of the eight forms this chord may assume, in connection with its " resolution " on the Core Triad, it may be noted that those containing Fah are of First Grade.

Those containing Fe are of the Second Grade, comprising " Supertonic Sevenths " and " Augmented Sixths."

The other forms of theoretically possible Envelope Tetrads are met with, but less frequently.

(It should be borne in mind that Re, frequently written Ma, is the Laxator Contra-determinator, and thus of First Grade.)

SUBSECTION 2

PLURAL CONCOMITANCE

Every tone inheres in a plurality of Series and consequently exhibits a dual valency with respect to the two converse species.

Among the many possible arrangements, there is only one matrical grouping which will enable the same seven tones to appear (liminally) in both species.

This is the arrangement of tones known as the Natural Scale, of which the two " modes " are determined by the position of the section terminators; these being the Hemicyclic Matrix, and its totally permuted (and commuted) equivalent.

The exhibition of relationship usually takes the form of an

expression of terms in rows. The three orders of relationships involved are reduced to two by the achromatic elimination of the zero; and if the first-order terms are written horizontally, and the second-order relations vertically, the two-dimensioned vision can grasp the aggregate as a whole.

The tertriadal aspect is put in evidence by writing every alternate row of terms so that they stand over or under the space between the terms of the next row, this member being the predominant or recessive determinator (permuted or not) with respect to the two terms above and below.

In E.T. the rows "recur" at every fourth diagonal term, owing to the equation PDDD = WP.

Since $5^3=125$, $2^7=128$, we are limited to three rows of uni-valent expressions.

The members of one row correspond with those in the row below three and a half steps away.

$$\text{TTT}=\text{LD, and LL}=\text{TD}$$
$$3^4=81,\ 5\times 2^4=80$$

The infinitely extending system of just intonation is thus limited by E.T. to the seven terms of the matrix.

The nominating quadrinomial is written down as a row of four terms with the three educt determinators above if fundamental, and below if coincidental.

It is convenient to add the alternative permuted determinators on the opposite side of the quadrinomial, thus showing the group of ten terms out of which the various forms of seven-toned matrix can be selected.

The commatic relationship between the rows, or rather the commatic difference of tuning between similar E.T. terms in different rows, has been indicated by various authorities by means of symbols.

Hauptmann introduced capital letters for the nominant quadrinomial, and small letters for the determinators.

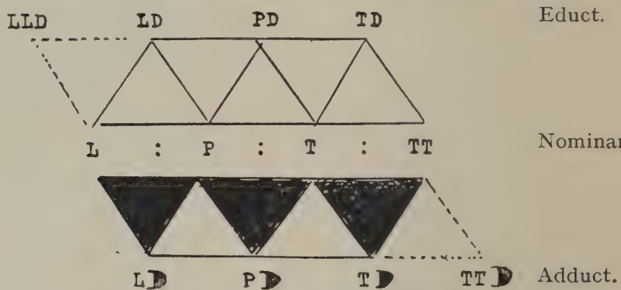
Helmholtz used the small lines drawn above or below suggested by Von Oettingen; Ellis, to save type-setting, substituted the marks of expression, † ‡.

For further information the works of these investigators should be consulted.

Jonquiere pointed out that by drawing lines to join up the

terms of a triad, the triangular symbol already in use (which has been ascribed to Andre) arose naturally, the apices representing the determinator, and the base being in the row of Pythagorean terms. The direction of the apex, upward or downward, represents the F or C species.

We illustrate this arrangement; it is to be noticed that the Nominant Quadrinomial is arbitrarily made Fundamental in species; a coincidental nominant would, of course, be inverted.



The upper figure represents a Hemicyclic Matrix, consisting of the three fundamental triads.

These are the predominant triads of that nominant species, but it is seen that there are two inverted triangles formed between them.

These are the two triads of Recessive Species, and upon extending the determinator Pythagoreans to form the quadrinomial by including LLD=TT=J (the Yoke of Concomitance) we obtain a third triad, which as part of the Envelope belongs to another class (Link).

Similarly, the lower figure represents a totally permuted matrix, with the three-permuted predominant, and two permuted recessives (dually permutable-Ortho-recessives) which are seen to be of the same species as the Nominant.

By extension to TT◗=L=$\frac{\pi}{2}$J (the Yoke of ortho-concomitance) we obtain the corresponding Link triad.

The figures thus form a general rhombus, of which the four corners constitute a homochrome, 3V or 3Y.

It is instructive to write the terms in Tonic-Solfa notation, on the " natural scale " basis.

Fundamental Nominant

Ray		Lah		Me		Te			Educt
	Fah		Doh		Soh		Ray		Nominant
		La		Ma		Ta		Fah	Adduct

Coincidental Nominant

Te		Fe		De		Se			Adduct
	Ray		Lah		Me		Te		Nominant
		Fah		Doh		Soh		Ray	Educt

The concomitant members of the hemicycle are those to the right of the dodecanal vertical diameter drawn from L to TD inclusive, Q^0+Q^1. The ortho-concomitant members are those above the horizontal diameter, drawn from J to its diametrical, Q^0+Q^{-1}.

To translate the concomitant to the ortho-concomitant involves a turn of quarter-phase in a definite direction.

The diagonal relationships upwards to right are infra second order (G, M); upwards to left are ultra second order (V, Y), being thus supplementary in mutual relationship.

To save the necessity of printing the triangular symbol (and also in speaking of same), the terms Delta $\triangle$ and Nabla ∇ could be used without, of course, any implication of their usual mathematical meanings.

We see that two adjacent rows are out of step by $3\frac{1}{2}$ grades, *i.e.* $7/12\ \pi$.

A complete cycle of any pair of Pythagorean rows presents the bi-dodecanal, which may be either Concomitant or totally permuted into Ortho-concomitant.

These considerations lead to an aspect in which the Hemicyclic Tertriads of concomitant species are linked together by the "Diminished Triad" formed by their mutual "link dyads" about J, viz.:—

$$\begin{smallmatrix}F\\C\end{smallmatrix}\binom{L}{TD} : J : \begin{smallmatrix}C\\F\end{smallmatrix}\binom{L}{TD}$$

This "diminished triad" is a homochrome (V+V or Y+Y), and therefore a Discord, but in many ways it may be regarded

and treated as analogous to a Triad, as will be seen in the section dealing with Progression; and by the fact that TD can be permuted to make it an objective triad.

The rules of Strict Counterpoint are designed to eliminate discords and directed chords, but even here the first inversion of the Link is admitted (Bundle Analysis, Y + V).

From the aspect of the Hemicycle in which J yokes the two concomitant quadrants of converse species, this chord may be regarded as the Link of a symmetrical grouping of triads, *i.e.* the predominant and recessive tertriads.

The tones of each tertriad form a " mirror image " of each other.

The two tensors differ by an Oscillant, and stand at the opposite ends of the mutual Pentad.

$$(9 : 7 : 3 : 5 : 1)\mathrm{CT}$$
$$\mathrm{FT}(1 : 5 : 3 : 7 : 9)$$

The pentad deprived of its Tensors, leaves the Link, which is formed by:—

$$(7 : 3 : 5)\mathrm{CT}$$
$$\mathrm{FT}(5 : 3 : 7)$$

The two primes are mutually determinative, and are symmetrical about J and its diametrical.

The two laxators stand in the extreme relationship W/2, so that ${}^{F}_{C}L = \pi\,{}^{C}_{F}L$.

This interval is unique in the hemicycle, although formulated between other terms in the composite tertriad.

It may be noted that when these two tones appear in successive chords in any other relationship than that of mutual inherence in the same envelope, the effect known as the " False Relation of the Tritone " arises. This will be discussed in Successive Tonality.

The recessive tensor triad is given by the members (3 : 7 : 9) of the Predominant Tensor Series.

The ortho-concomitant recessive laxator triad is given by the same members of the Predominant Laxator Series.

Ortho-concomitance is exactly concomitance turned through half a period in phase in the definite direction (F laeval) (C dextral).

The Laxator becomes the new Yoke, and may be written $\frac{\pi}{2}J$.

The Terquadrant formed by a nominant quadrant with its concomitant and ortho-concomitant, forms an "Envelope" about the Diametrical Quadrant.

The four aspects of a quadrant

(Predominant – Recessive) × (Unpermute – Permute)

embrace the twelve tones of the E.T. dodecanal.

This system of relational nomination is, of course, only one of the many presented by the formal connections of tonality, and care should be taken to prevent any of the conventional ideas of "near" and "far" relationship being imported from the arbitrary conditions of notation and keyboard.

At the same time, the considerations upon which the system is based are of sufficient importance to merit the somewhat lengthy exposition.

These theoretical arrangements may appear to be but of little beyond academic interest, but they throw an interesting light upon the old nominalist controversy as to which is the true "minor" of a given "major" tonic or relative.

We now see that they are both "Minor" in the sense of being similar subordinate relations on the right and left hands.

The three quadrants form the Terquadrant $Q^{-1} : Q : Q^{1}$ whose "poles" are each orthogonal to the nominant quadrant, and mutually opposed in phase.

Similar conditions apply to the converse question as to the "major" of a given Minor key.

It is also evident that a given tone is bivalent, since it inheres in both predominant and recessive species.

The tone "Doh," for instance, has a predominant impression of FP and a recessive aspect as CPD in the Major Key.

When this duality is recognised, the controversy as to the correct way of indicating the minor scale in Tonic-Solfa terms is seen to be beside the mark, or at any rate, the question becomes one of mere technical convenience.

The Recessive triads of a given matrix take the form:—

$$\binom{\mathrm{D}}{◗} : \genfrac{}{}{0pt}{}{\mathrm{T}}{\mathrm{P}} : \binom{\mathrm{D}}{◗}$$

thus having the appearance of a triad which has been "turned inside out."

Such a configuration of tones is not found in the early harmonic series, but the chord is acoustically just as much a triad as the predominant form.

In a system of tonal determinance the distinction is evident, and in the analytical aspect recessive triads are recognised as Compound chords, *i.e.* as Discords (Envelope members) or as Seminomial chords of transition between Predominant triads.

The system of concomitant and ortho-concomitant (direct and permuted) triads enables every triad of the dodecanal, except the three diametricals of the trinomial, to be named.

Of these latter, the two polar triads appear in the Ter-heptad or Quintriadal grouping, leaving the triad of the Diametrical Prime (Diacentron) as the sole "excluded" chord of the aggregation.

This is the triad of the Sexatensor and Sexalaxator, and is obviously the only chord ambiguous in polarity.

In a given seven-toned Matrix there are therefore:—

Three Predominant Triads, in which one tone is permutable.

Two Recessives, dually permutable.

Two Links, of which one tone is permutable.

The two Links are derived by splitting the Envelope into:—

$$\mathrm{T}^{\mathrm{D}}_{◗} : \mathrm{TT} : \mathrm{L}$$

$$\mathrm{TT} : \mathrm{L} : \mathrm{L}^{\mathrm{D}}_{◗}$$

They are concordant triads when in the form:—

T◗ : TT : L
TT : L : LD

and homochromes when:—

TD : TT : L
TT : L : L◗

The "Natural" homochrome Link is thus:—

TD : TT : L

which is common to both predominant and recessive concomitant species. Since its terminals are interchangeable, we get, as the possible forms, the two chords:—

Fundamental	Ta : Ray : Fah
Coincidental	Te : Ray : Fe

The other Link is the Ortho-concomitant, and its two consequent forms are:—

Coincidental	Ra : Fah : La
Fundamental	Ray : Fah : Lah

The two strange chords in the above expressions are the Diametrical Poles, forming part of the Parasyntonic Envelope:—

With respect to Doh	(Fe : Ray : Te : La : Fah : Ra)
With respect to Me	(Re : Te : Se : Fah : Ra : Ta)

The contra-determinators of the nominant quadrinomial LⱯ, PⱯ, TⱯ, and TTⱯ, are equivalent to P◗, T◗, L, and P respectively.

It may also be pointed out that a set of rows of Pythagoreans whose vertical relationships are of contra-second order can be written down. In this case the first-order intervals are conveniently read as R, and the contra-determinators above and below, forming triangles, are the F and C series bisectors.

The system of two dimensions, of course, lends itself to the exhibition of any pair of independent relationships (orders) of tonality, and if the paper be considered as folded at right angles along the centre line, the idea of orthogonality can be still further visualised.

SUBSECTION 3

CHORDAL SYSTEMS

Consideration may now be directed towards the nomination of chords within (conserved) and with respect to (translatory) a given matrix.

The Triad is taken as the unit simple element.

Any chord composed of parts of different triads may be generally known as Compound.

The term " Chord " may be applied not only to the complete

group of possible tones, but also to any portion which definitely represents its chord, the other members suggested by the environment being regarded as "Latent" in sound, like the silent beats in rhythm.

In this way, Dyads, and even Monads, may be regarded as chords.

Portions of compound chords of which the actual tones form concordant triads, are nevertheless to be regarded as Compound.

An analytical view of the matrix presents various groups of chords, which may be allocated into classes of Complementary, Antithetical, and Mixed.

The student of empirical harmony will recall groups, varying in name according to authority consulted, but recognisable in actual musical manifestation. These are:—

(1) Triads on the degrees of Scale.
(2) Tonic, Dominant, and Supertonic Sevenths, Ninths, Elevenths, and Thirteenths (Major and Minor).
(3) Chromatic Discords.
(4) Augmented Sixths.

These are definite enough to pass current in musical terminology.

One of the early problems presented to the student of empirical harmony is the definition of chords in a key.

Regarding any given group of tones as a chord, it may possibly be named with respect to several different loci; thus, for instance, the tensor triad is prime in one key, and laxator in another, recessive in another, etc.

The criterion is afforded by the aspect of the Matrix developed from the Tertriad and Tri-heptad, in which the aggregate is divisible into the sub-groups of Core and Envelope.

The Core is the definite Triad of the Prime, the Envelope is a Hexad composed of the two polar triads, which when presented separately involve the latent members of the opposite polarity.

This is equation about a Centron, and constitutes the principle of Conservation.

The principles of Matrical Limitation determine the extent of inherent grouping.

There are, however, other aspects, which it will be advisable to consider.

The Core is a singly permutable triad, but the Envelope is doubly permutable.

Between the three predominant triads are formed two " recessive " chords, which may appear similar to triads; but which, in the conserved aspect, are actually compound, being Seminomials, composed of part of the Core, and one pole.

They take the general form of a triad which has been turned inside out, since they contain two determinators, with a Zero first-order tone as " mediant."

Unlike the Predominant Triads, which remain concordant when permuted, these Recessives can be Discords.

The two determinators are independently permutable, thus presenting four varieties of chord.

Chord.	F Example.	C Example.
LD : P : PD	Lah : Doh : Me	Soh : Me : Doh
L◗ : P : PD	La : Doh : Me	Se : Me : Doh
L◗ : P : P◗	La : Doh : Ma	Se : Me : De
LD : P : P◗	Lah : Doh : Ma	Soh : Me : De

presenting two exact triads, educt and adduct, together with an augmented and a diminished triad.

The recessive triad can only appear as a compound in one locus, although it may be a concord.

Regarded otherwise, it is the result of translation, concomitant-commutation or otherwise.

It may also be regarded as bi-located, as a chord at the instant of translation from one locus to another.

The form P : PD : LD is equivalent to the tetrad P : PD : T : P◖ with the tensor latent, which is treated progressionally as a tetrad.

We now come to the two " Link " triads which form part of the Envelope Tetrad, viz.—

$$\mathrm{T}^{\mathrm{D}}_{◗} : \mathrm{TT} : \mathrm{L}$$

$$\mathrm{TT} : \mathrm{L} : \mathrm{L}^{\mathrm{D}}_{◗}$$

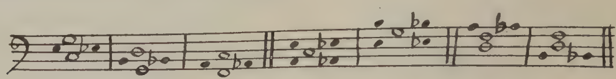
Each of these is " permutable," presenting one form identical with a concordant triad, the other being a triple-violet or triple-yellow homochrome, the extreme interval of which (in close position) is a Semi-octave.

Thus, in E.T.:—

T◗ : TT : L = LL (P : D : T)
TT : L : LD = TT (P : ◗ : T)

the test being the univalence of the quasi-determinator, *i.e.* the "mediant" tone.

The three predominant, two recessive, and two link chords, thus present seven three-toned chords, which among their variants constitute seven triads.



Predominant. Recessive. Link.

The conserved chordal system has another aspect, properly approached from the side of Successive Tonality, viz., that presented to the student in Counterpoint, who takes up a somewhat different standpoint from the harmonist; regarding the Scale of seven tones (any mode) as the basis, the triad being merely considered as the coherent expansion of a scale-note into a group of three.

The seven triads thus given are dependent upon the actual scale and mode chosen.

Taking the natural scale as basis, it is seen that the triads presented comprise three F, three C, and one diminished.

This applies to both species, and is therefore valid in the Major and the Declinear Melodic Minor, as well as in the so-called Ecclesiastical Modes determined from the "Natural" (Hemicyclic) Scale by selection of terminating limits.

The Harmonic Minor (Symmetrical, with C Prime Triad) presents two C, and two F, triads, two diminished (Link Chords) and one Augmented (recessive).

The Aclinear Minor (Composite) presents one C, three F triads, and three "Links."

The triads are treated as notes of a scale; and the concordance of interval with effect of inversion determines the applicability. These matters do not primarily concern the chordal criterion based upon the Tertriad.

The strict contrapuntist cannot avoid including one form of link chord in the system, although by his own rule all discords are eliminated with a view of obtaining freedom from "directed

progression;" the idea being to develop resource in the treatment of limited material by discipline and restraint.

The chord in question (the first inversion of the Link Triad) is, however, resolved upon "bundle analysis" into two concords, Y plus V, so that by neglecting the secondary discord of the tritone, it can be said to comply with the rules of the method.

The relative concordance of each otherwise indifferentiate triad of similar species is tested by the addition of a Tertriadal Axis; *i.e.* the Prime (P, LT) and the Tensor (T, PT) which are known as Pedals or Organ-points.

The Dyads are measured in grades according to the empirical classification dating from Grecian (and possibly before) times, which are confirmed by Stumpf's measurement on the basis of average perceived synergetic fusion.

Zero Order	Z, W	Absolute concordances
First Order	B	Perfect concordance
,,	R	Ditto; except when in the "Bundle," when it is regarded in the Fluent aspect as a Discord.
Second Order	G, V, Y, M	Imperfect concordances
,,	Tritone W/2	Least Discordant
Major Second / Minor Seventh	O, (W – O)	Medium ,,
Minor Second / Major Seventh	A, (W – A)	Most ,,

Resolution consists in progression from a greater to a lesser discord, or conversely, from a lesser to a greater concord.

The "Tonic" pedal is the latent member with respect to the Core component of the Matrix.

The "Dominant" pedal performs the same function with regard to the Envelope Component, and the two together act as the conserving latents with respect to the whole of the Matrix.

Although the two axes are the primary and most evident "pedal points" it is possible to utilise not only the note but the whole or part of the chord. This gives rise to secondary forms of pedal, comprising the remainder of the Core and Hexad Envelope.

In this way the whole Matrix may be used as a conservant "pedal," and may be applied in the form of a scalar "counter-

point " or many other forms. The actual guiding principle of practice is applicability as regards clearness, especially as regards predominance of concomitant species. This latter condition usually rules out second-order tones as axes, but not invariably.

These matters are properly to be considered under the heading of Successive Tonality, but the basic principle is dealt with here in order to show the method of " Latency " by which chords are distinguished as conserved from the matrices of other loci.

The tertriadal aspect involves that the polar triads shall always be regarded as part of the whole Envelope.

This may be practically tested by sounding such of the latent tones as the progression permits.

Generally, a laxator triad may be turned into an " Added Sixth " (so-called chord of the Eleventh) and a tensor triad into a Dominant Seventh, without altering the effect of the progression; but if translation to either polar locus has really occurred, the latent tones cause " re-translation " to the original or a further locus (Modulation).

In the same way, the test of conservation is the application of a Pedal.

P is the test of a Core, T that of the envelope, and (P : T) that of the locus of the whole matrix.

The other tones act as " weaker " pedals, but L, when its inherence in the envelope is evident, is not infrequently used for short passages. By itself, L is the inverted Tensor and thus belongs to the opposite species, so that a true " Subdominant Pedal " is rare.

The extreme case of a Heptad pedal is rarely met with, and requires special treatment in which Rhythm, etc., conditions may enter.

The criterion of conservance thus involves cognisance of Latency, whereby the chords belonging to a given matrix can be determined.

In actual practice, we are also concerned with the effect produced as apart from its rational tonality. This renders analysis a somewhat complicated matter, but the general principle applies.

In the foregoing remarks, attention has been restricted to triads, and the contradeterminator has not been considered as an independent element.

The inclusion of this tone extends the material, but does not invalidate the principle.

The radical tetrads resemble the envelope of the laxator locus, and, in general, the liminal approximatability of radical and envelope tetrads introduces pathways of tonal manifoldity, which often look in notation more complicated than they sound.

As a general principle, anything that the ear accepts has a rational basis, but this may not always correspond with what is arbitrarily or conventionally regarded as simple.

The only two dodecanal triads which are—as triads—definitely excluded from the Locus, are those of the Diametrical Prime and its Permute.

These are neutral in polarity (in E.T.) and form a class by themselves.

As compounds, their constituents may appear, and they would also be included in Just Intonation as extreme polar forms, in which case (Fe : Le : De) would be differentiated from its E.T. equivalent (Sa : Ta : Ra), Le and Ta being replaceable by Lah and Taa.

But one cannot enjoy the advantages of E.T. generality and at the same time neglect its logical consequences.

Consequently, the Diametrical Core Triad and its Permute are regarded as Anti-matrical and inadmissible as chords of a "key."

Their occurrence in practice is generally due to the use of the incongruity for the sake of some special effect.

The Triad chords of a Matrix can be examined from the arrangement of tones as follows:—

T	
P^D_◗	Core Triad
P	Semi-laxator Recessive
L^D_◗	Laxator Triad
	Laxatorial Link
L	
TT	Tensorial Link
T^D_◗	Tensor Triad
T	Semi-tensor Recessive
P^D_◗	Core Triad
P	

By taking the Tri-heptad, the seven triads are extended into a nominal twenty-one, reducing by axial redundancy to the original seven, plus:—

The Bi-laxator triad . .	LL	: $LL^{D}_{\mathbf{D}}$	: L
Semi-bilaxatorial recessive	$LL^{D}_{\mathbf{D}}$	: L	: $L^{D}_{\mathbf{D}}$
Bi-tensor triad . . .	TT	: $TT^{D}_{\mathbf{D}}$	: TTT
Semi-bitensor recessive .	$T^{D}_{\mathbf{D}}$	: TT	: $TT^{D}_{\mathbf{D}}$

Pseudo-seminomial triads are also formed, but need not be now considered.

The Tertriadal Matrix also presents three Tetrads, two " Axis " and the Envelope, together with four " intermediate " forms:—

T
$P^{D}_{\mathbf{D}}$
P
$L^{D}_{\mathbf{D}}$
L
TT
$T^{D}_{\mathbf{D}}$
T
$P^{D}_{\mathbf{D}}$
P

Second Primal Tetrad
First Primal Tetrad
Prime Axis Tetrad
Envelope Tetrad
Tensor Axis Tetrad
First Tensorial Tetrad
Second Tensorial Tetrad.

We thus see how the chords of a given matrix appear as part of a general group allocatable either as Core, Envelope, or Compound.

The grouping constitutes the conservance, and nomination depends upon, and therefore involves, recognition of the latent members.

A seven-toned Matrix contains seven three-toned chords comprising:—

(*a*) Three permutable chords; each having two forms, viz. one core triad, and the two polars of the envelope.

(*b*) Two recessive or semi-nomial chords; being the semi-tensor and semi-laxator triads, which are linked by the two axes P and T respectively to their nominant.

These contain two permutable determinators each, and can thus appear in four forms, viz. two concordant triads, F and C, one "augmented," and one "diminished."

(*c*) Two Link chords, totally inherent in the Envelope; which may be termed (from preponderance of elements) tensorial and laxatorial respectively. These each contain one determinator and can therefore appear in two forms; one a concordant triad, the other a homochrome.

There are thus eighteen forms of three-toned chord in the Matrix, of which twelve are concordant triads (out of the twenty-four possible in the Dodecanal).

Of these, seven are co-matrical, the remaining five alternative forms in a "natural" matrix, the proportions varying in the "composite" varieties.

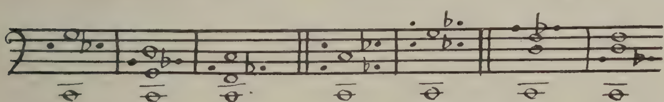
The hexad envelope presents three tetrads and two pentads; while the whole matrix, as a Heptad, presents four tetrads, three pentads, and two hexads.

The theory of the Latent Axis enables a distinction to be made upon the basis of relative concordance.

The two groups may be illustrated (in F species) by Staff notation:—

PRIME AXIS (CORE GROUPING)

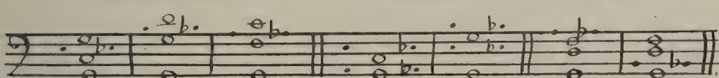
Predominant. Recessive. Link.



Con. Dis. Con. Con. Dis. Dis. Dis.

TENSOR AXIS (ENVELOPE GROUPING)

Predominant. Recessive. Link.



Con. Con. Dis. Dis. Con. Dis. Dis.

Anticipating the progressional aspect, it is seen that resolution, in the contrapuntal sense as illustrated above, consists in

eliminating an intrinsic or latent discord by the progression of a chord-tone to a concord.

This may be compared with the harmonic or chordal sense, in which resolution implies the progression from an envelope to a core, or *vice versa,* according to the " latent axis "; and refers secondarily to a simplification of the envelope by convergence upon the polar foundation, *i.e.* by contraction to a Tensor Tetrad under Vertication conditions.

Before proceeding to the general examination of chords, it is necessary that there should be agreement upon the definition of the term " Chord "; otherwise ambiguities of terminology easily arise.

The physical aspect of a Chord admits of no uncertainty. A chord is a simultaneous presentation of tones notationally shown by vertical grouping, and actually a vector sum of the components of vibration.

Physiologically, a chord appears as a compound sound, *i.e.* as a stimulation of more than one region of the basilar mechanism. As such, it cannot be distinguished from portions of harmonic series. The difficulty is appreciated when trying to analyse a phonographic record.

Psychologically, a chord assumes an altogether different aspect, since it is necessary to consider how chords come to be, and this involves recognition of time in relation to a progression.

Here a distinction appears between the true chord as an entity, and any coexistent colligation of tones due to polyphony.

An acoustical definition of the term chord thus appears to transcend the domain of inquiry. However formulated, such a definition can but appreciate a chord as a sound whose components can be analysed and specifically described.

But the tonal conception of a chord is essentially different, since it involves nomination of members on a more or less definite basis.

We may have, at any one epoch in time:—

(*a*) Tones of one chord coexisting.
(*b*) Tones of part of one chord coexisting with tones of another chord.
(*c*) Tones of a chord coexisting with tones of a scale.
(*d*) Tones of scales coexisting with others.

And, generally, any variants and extensions of the above.

In experience, these groups are not always clear-cut in classification, but merge into each other, the criterion of distinction being the predominance of the defining chords, scales, etc.

As every observer can see, it is possible to divide coexisting tones into:—

(*a*) Harmony groups of one chord.
(*b*) Suspensions, Anticipations, Retardations, and one aspect of Pedals.
(*c*) Passing notes, Changing and Auxiliary notes, and Grace notes.
(*d*) Pure polyphonic colligation.

The principle of Latent Components paves the way to a comprehension of tones which "carry themselves over chords."

These may be known as "Autophors." Again, the seminomial chords exhibit a case in which part of one chord progresses to another chord, while the other part remains, thus presenting a "halfway" time stage.

The present consideration is devoted as far as possible to what may be called Harmony Notes, *i.e.* those forming definite chords.

The actual way in which these chords are presented, whether *en bloc* or distributed in time, is not now under consideration, nor is the achromatic arrangement, etc.

These are important matters in the empiric theory, and will have to be ultimately brought under consideration, but at present it is the abstract concept of a "Chord" that is the matter of discourse.

The purely tonal view differs also from the musical, in that the appropriate use of any chord does not concern us.

The view is admittedly an abstraction, and the materia musica is only rational in the whole of manifestation.

It is obvious that the tonal and chromal members of the chordance system possess a plurality of nomination.

The standard is based upon predominance of audentity in any particular aspect.

Within the domain prescribed by the E.T. the distinction between Conserved and Translative chords has to be drawn.

A conserved chord has a static effect upon the locus of its matrix.

At the instant of translation into another matrix, some aspect

of its possible bivalence becomes predominant, the audentity of the original locus being diminished, and that of the new enhanced, by various devices familiar to practical workers.

The system of a matrix, as a source of chords, is exhibited by its tertriadal aspect, comprising the Core and the Envelope of the polar triads.

The core is always a simple triad of independent species, and consequently can be permuted without reference to the envelope.

The envelope appears as a Tetrad plus the two axes P and T, and is thus a Hexad.

Its two determinators are independently permutable with respect to each other and to the Core.

The Seriopolar aspect of the Matrix shows how the series of the polar tones can represent a kind of inverse tertriad, in which one polar triad predominates, the other being formulated by the higher members, while the " Core independent " (the Prime Determinator) appears as a very extreme presentation, *i.e.*:—

L_{15}=PD L_7=P◗ T_{13} approximates PĐ

The Envelope Hexad plus one of these tones presents a Heptad Matrix, nominated originally by the polar prime, but referable, in consequence of its quasi-tertriadal aspect, to the Centron formulated among its members.

The Trinomial

L : P : T

The Tertriad or Heptad

L Pole.		Centron.		D Pole.
P	:	T	:	TT
LD	:	PD	:	TD
L	:	P	:	T

which may be written in three columns of " local " order as follows:—

Zero.		Second.		First.
L	:	LD	:	LT (P)
P	:	PD	:	T
T	:	TD	:	TT

THE TRI-HEPTAD

L Heptad.		P Heptad.		T Heptad.
T		TT		TTT
PD		TD		TTD
P	——	T	——	TT
LD		PD		TD
L	——	P	——	T
LLD		LD		PD
LL		L		P

which is identical with the Quintriad.

THE QUINTRIAD

L		P		T		TT		TTT
LLD	·	LD	·	PD	·	TD	·	TTD
LL	·	L	·	P	·	T	·	TT

It is possible to extend both methods, so that we get Ter-triheptads, etc., as well as Septriads, Nonatriads, etc.

The principle of limitation to the Conserved Matrix may be expressed in hemicyclic terms by the Phase tangent (see previous chapters) but is actually based upon the maintenance of the Core as centron.

This principle, and its logical result, is evident upon considering the plurivalence of each member in the E.T.

In the Triheptad, the interval TTD : P=W/2 is presented. This is antinominous with the first-order chromes of the Polar triads.

In the Ter-triheptad or Septriad the relation TTTD : LLLD =T(W/2) is presented by the determinators of the extreme triads. This is the first-order chrome of the Core Triad with P displaced to AP, a direct antinomy, which bars the recognition of the group as a Matrix.

The existence, in the same group, of a Core triad and its Diametrical, is a contradiction of terms.

The Core triad is Centron, and in respect to same any other triad is either on one or the other polar side of it.

Compound chords can always be analysed into such simple components.

The Diametrical is rational in consequence of the evanescence

of polarity at the sixth Pythagorean step. Hence the condition which nominates the Core as Centron vanishes at this point, and the locus of the whole system becomes ambiguous.

The centroid of a mechanical system exists because of the equilibrium of the parts of such system about it. If the latter becomes ambiguous, the centroid is necessarily unstable.

To push the argument to its logical conclusion, the coexistence of a Centron and its Diametrical implies that the centron nomial, tone, dyad, triad, chord, or whatever it may be, is replaced by the compound comprising it with its similarly equilibrated diametrical.

This, in the case of the tone nomial, is the semi-octave; a dyad which stands in highest contrast to the harmonic first-order chromes B and R of which it is the mean.

The whole or part of the "chord" formed by the two triads exhibits the same effect, viz. that of Antinomy.

Thus a definite limit is imposed upon chordal systems by the determinate tonality possible with the E.T. system, and the Triheptad (Quintriad) remains the frontier.

The Triheptad forms a matrix embracing the "non-cadential" Interheptads, chords which are known in the Fundamental form as Augmented Sixths.

The Diametrical aspect of these chords will be considered later, but here it is to be noted that there are two forms, viz. containing notes selected from the Heptads of:—

(1) L and P (2) P and T

Taking the Quintriad, we have for No. 1:—

German Form (Recessive Triad) (LL◗ : L : L◗) + TD
French Form . . . . (LL◗ : L) + (T : TD)

And for No. 2:—

German Form . . . . (L◗ : P : P◗) + TTD
French Form . . . . (L◗ : P) + (TT : TTD)

The corresponding interpolar form (3) of L and T is not common in experience.

German Form[1] . . . (LL : LL◗) + PD + TTD
French Form . . . . (P : PD) + T◗ + TTD

[1] It is seen in Beethoven's B flat Piano Trio resolving on the Laxator Triad.

This is obvious from the natural antinomy and excentric form, since the group comprises components of the Prime and Diametrical Triads.

Coincidental Interheptads are possible, but owing to the extremity to which coherence must be pushed against vertication, are not so evident as Fundamental.

They are, however, not so rare in the classics as might be imagined.

The Triheptad (Quintriad) may also be regarded as constituted of a Centron (Core) triad, its Primary and Secondary Envelopes.

These latter are the envelopes of the polar triads, and the group could be written $E^0+E^1+E^2$.

We have now briefly surveyed the whole system of chords inherent in a Matrix, from both the Tertriadal and the local nominance aspects.

This is not a treatise upon harmony, therefore illustrations of the chords and their use and abuse, together with their evolution and possibilities, are not given.

The reader may be somewhat puzzled by the references to the contra-determinator in a system of chordance which purports to exclude it.

As a matter of fact, the tone is just on the boundary and is of some considerable audental importance.

It is logically excluded by the E.T. system, and therefore is not found in either Staff or Solfa notation, nor upon the claviature of ordinary instruments.

But the E.T. system itself is only an imperfect compromise, determinate though it may be; consequently it is of some advantage to consider the contra-determinator as an independent member while basing the survey upon the E.T.

It must be remembered that the next odd series member, the ninth, is compound (3×3), so that a considerable gap extends between 7 and 11, the latter being much less audentally evident on the average.

The permuted contra-determinator has a real basis in the fundamental species. The seventh member of the harmonic series, given by a bell-mouthed wind instrument, can be flattened by the insertion of the hand to the nearest lower E.T. note.

Thus FP◖=Lah, FT◖=Me, FL◖=Ray.

By " blowing the note sharp " it is sometimes possible to force the member up to the E.T. unpermuted values, viz. Ta, Fah, and Ma.

It becomes thus possible to obtain both extreme tones of " Sevenths " and " Added Sixths " on such instruments, although it must be admitted that the quality of the sound suffers.

It may be noted that 4(P : Ɑ)=P(2G+B+5W) very nearly, or less closely to 9B, so that 2(P : Ɑ)=4½B.

It may also be noted that LⱭ=CPⱭ, the ratio being 48 : 49 (Degree of Approximation, 5½W).

Regarding the Laxator as the Commute Tensor of the Prime, the predominance of the " Chord of the Added Sixth " is evident from the quasi-identity; *i.e.* the Radical Tetrad of the Laxator, and the Commute Radical Tetrad on the Prime.

The question as to the possible future recognition of the interval (P : Ɑ), and the contra-determinator tone, as distinct entities, may occur to the reader; but it is seen that the conditions under which we enjoy the advantages of the E.T. system practically bar any such recognition.

That the Chrome is being recognised and used in practice, no one who is well acquainted with the modern trend of harmony can well deny; but it has not as yet received the benison of empirical theory, or a place in notation.

Should a nearer approximation to just intonation come into use, or even a quarter-tone system be found practicable, it is noticed that the Prime Bicontra-determinator, frequency 49, bisects the interval between the Tensor 48 and the Bideterminator 50.

It is a conceivable aspect of the justly intoned " minor thirteenth," appearing as LⱭ when L=TⱭ.

Finally, a word may be said as to the implied influence of the Tetrad upon Polyphony.

From the fact that the Conservative members at any epoch in polyphonic presentation consist of members of a triad (3), and its tetrad envelope (4), without the two axes bringing the total up to six, it would appear that to maintain a continuous number of independent parts, four would be the most general economical number.

Tetraphony is thus foreshadowed as the standard of procedure.

Again, since the envelope tetrad is divisible into two opposed poles or species, its association with balanced two-part contrary motion (enantio-phony) is obvious.

The duality of phonal direction is most important, and more will be said on it later.

The case of the four "floating" tones of the envelope Tetrad, which can occupy any pitch except those within the liminal range of the two cores (and which are represented by the four tones of the Symmetrical Tetrad), presents a direct principle of limitation based upon the Tertriadal aspect of the Matrix (which curiously resembles the Hexachord system of Guido d'Arrezo).

In this there are three Triads, with their Three Symmetrical Envelopes, as the material; and limitation to a matrix follows from the E.T. coincidence of tones.

Thus the Matrix appears (Fundamental example):—

Laxator.		Prime.		Tensor.	
Triad.	Env.	Triad.	Env.	Triad.	Env.
Doh	Ra	Soh	La	Ray	Ma
Lah–La	Ta	Me–Ma	Fah	Te–Ta	Doh
Fah	Soh	Doh	Ray	Soh	Lah
	Me		Te		Fe

A simple tone nominating system is thus presented by this view.

SUBSECTION 4

ANTINOMINANT SYSTEMS

It is possible to develop an Antinominantal chordal system, by triadal extension upon the nomials of the antinominant cycle, in a similar manner to the method adopted in the case of Pythagorean groupings already discussed.

The important difference between the two groupings must always be borne in mind.

The Pythagorean grade is chromal and coherent, being itself one of the components of a triad; so that the matrical system developed from it presents certain immediate arrangements of simultaneity.

(As an extreme case in which the F.T. Heptad can be presented euphonically, we may have:—

$T(1 : 3 : 5 : 6 : 7 : 9 : 13)$, T_{11} being deleted to avoid collision with T_5.)

The Antinominant cyclic grade is Fluent; and thus inadmissible as a chordal element.

The rationality of its use depends upon the succession of tones so near in pitch as to be influenced by each other's tract, thus presenting an approximation to the scalar continuity of identity in pitch.

Thus, although groups may be constructed similar to those of the Pythagorean system, it is evident that they will not represent direct chordal forms, but modifications thereof, suitable to non-simultaneous presentation and colligation.

Although the subject of progression is deferred for consideration to a later chapter, the antinominant cycle relationships may be conveniently examined now, with a view to the comparative aspect, in which they appear in regard to chordal development.

The Antinominantal " Chordance " is to be regarded as the formulation of a " clinear " extension of nomials, the terms being the concordant triads.

Looking at the aggregation at right angles, it appears as constituted by three parallel (or rather concentric) antinominant cycles of tone-nomials, each ring being of different order.

Upon proceeding outward from a given Nominant, the nomial chords approach more and more toward the Pythagorean forms.

This is obvious, since $PA^5 = PT$, and $PA^{-5} = PL$.

About half-way between, there is a region of commergence, in which the transition chords are found.

Although constituted on quite an independent basis of aggregation, it is convenient to approach consideration of the Antinominant Cycle from the already familiar Pythagorean; by substituting for each alternate term of the latter its corresponding " diametrical."

This may be done in two ways, viz. to the " odd " or " even " terms, so that there are two A forms collateral with every Pythagorean, and *vice versa.*

The primary form is that in which the Primes are identical, *i.e.* having a common nominant, so that the odd-numbered dial

terms are diametrically transposed, and when the nomials are triads, the whole chord is thus affected.

In Just Intonation, the process of extension is theoretically unlimited, but the E.T. closes the dodecanal cycle.

The following are the expressions of general form:—

The Trinomial

Even	πL	:	P	:	πT
Odd	L	:	πP	:	T

The Tertriad or Heptad

$$\text{Even}\quad \pi\left\{\begin{matrix} P \\ L^{D}_{\blacktriangleright} \\ L \end{matrix}\right\} \;:\; \left\{\begin{matrix} T \\ P^{D}_{\blacktriangleright} \\ P \end{matrix}\right\} \;:\; \pi\left\{\begin{matrix} TT \\ T^{D}_{\blacktriangleright} \\ T \end{matrix}\right\}$$

$$\text{Odd}\quad \left\{\begin{matrix} P \\ L^{D}_{\blacktriangleright} \\ L \end{matrix}\right\} \;:\; \pi\left\{\begin{matrix} T \\ P^{D}_{\blacktriangleright} \\ P \end{matrix}\right\} \;:\; \left\{\begin{matrix} TT \\ T^{D}_{\blacktriangleright} \\ T \end{matrix}\right\}$$

Similar constructions apply to any further form of Nomial grouping.

The Quintriad also appears as Odd and Even.

We may now give the actual form taken by these configurations in Tonic-Solfa Symbols.

Trinomial

F Species:—

Even	Te	:	Doh	:	Ra
Odd	Fah	:	$\binom{\text{Fe}}{\text{Sa}}$	:	Soh

C Species:—

Even	Fah	:	Me	:	Re
Odd	Te	:	$\binom{\text{Ta}}{\text{Le}}$	:	Lah

The Tertriad or Heptad

F Species:—

$$\text{Even}\quad \left\{\begin{matrix}\mathrm{Fe}\\ \mathrm{R}^{\mathrm{e}}_{\mathrm{ay}}\\ \mathrm{Te}\end{matrix}\right\} : \left\{\begin{matrix}\mathrm{Soh}\\ \mathrm{M}^{\mathrm{e}}_{\mathrm{a}}\\ \mathrm{Doh}\end{matrix}\right\} : \left\{\begin{matrix}\mathrm{Fah}\\ \left(\begin{matrix}\mathrm{Fah}\\ \mathrm{Me}\end{matrix}\right)\\ \mathrm{Ra}\end{matrix}\right\}$$

$$\text{Odd}\quad \left\{\begin{matrix}\mathrm{Doh}\\ \mathrm{L}^{\mathrm{ah}}_{\mathrm{a}}\\ \mathrm{Fah}\end{matrix}\right\} : \left\{\begin{matrix}\mathrm{De–Ra}\\ \mathrm{Le–Ta}\\ \mathrm{Fe–Sa}\end{matrix}\right\} : \left\{\begin{matrix}\mathrm{Ray}\\ \mathrm{T}^{\mathrm{e}}_{\mathrm{a}}\\ \mathrm{Soh}\end{matrix}\right\}$$

C Species:—

$$\text{Even}\quad \left\{\begin{matrix}\mathrm{Ta}\\ \mathrm{R}^{\mathrm{a}}_{\mathrm{ay}}\\ \mathrm{Fah}\end{matrix}\right\} : \left\{\begin{matrix}\mathrm{Lah}\\ \mathrm{D}^{\mathrm{oh}}_{\mathrm{e}}\\ \mathrm{Me}\end{matrix}\right\} : \left\{\begin{matrix}\mathrm{Se}\\ \left(\begin{matrix}\mathrm{Te}\\ \mathrm{Doh}\end{matrix}\right)\\ \mathrm{Re}\end{matrix}\right\}$$

$$\text{Odd}\quad \left\{\begin{matrix}\mathrm{Me}\\ \mathrm{S}^{\mathrm{oh}}_{\mathrm{e}}\\ \mathrm{Te}\end{matrix}\right\} : \left\{\begin{matrix}\mathrm{Ma–Re}\\ \mathrm{Sa–Fe}\\ \mathrm{Ta–Le}\end{matrix}\right\} : \left\{\begin{matrix}\mathrm{Ray}\\ \mathrm{F}^{\mathrm{ah}}_{\mathrm{e}}\\ \mathrm{Lah}\end{matrix}\right\}$$

The Seriopolar aspect may be given.

Series of Each Pole

$$\mathrm{F}\pi\mathrm{T}(1 : 5 : 3 : 7 : 9 : 11 : 13 : 15)$$

$$\mathrm{Ra} : \mathrm{Fah} : \mathrm{La} : \begin{matrix}\mathrm{Te}\\ \mathrm{Ta}\end{matrix} : \mathrm{Re} : \begin{matrix}\mathrm{Sa}\\ \mathrm{Soh}\end{matrix} : \mathrm{L}^{\mathrm{a}}_{\mathrm{e}} : \mathrm{Doh}$$

$$\mathrm{F}\pi\mathrm{L}(1 : 5 : 3 : 7 : 9 : 11 : 13 : 15)$$

$$\mathrm{Te} : \mathrm{Re} : \begin{matrix}\mathrm{Fe}\\ \mathrm{Sa}\end{matrix} : \mathrm{L}^{\mathrm{ah}}_{\mathrm{a}} : \mathrm{Ra} : \begin{matrix}\mathrm{Me}\\ \mathrm{Fah}\end{matrix} : \mathrm{S}^{\mathrm{oh}}_{\mathrm{e}} : \mathrm{Ta}$$

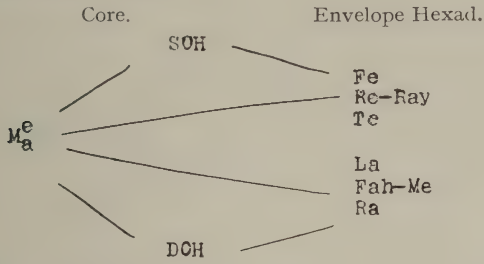
Putting $\pi\mathrm{T}\reflectbox{D}=\pi\mathrm{L}$ as in the Pythagorean case, we have the vertical "Seriopolar" of two Tetrads:—

$$\left\{\begin{matrix}\mathrm{F}\pi\mathrm{T}(1 : 5 : 3 : 7) & \\ & \mathrm{L}(1 : 5 : 3 : 7)\end{matrix}\right\}$$

$$\left\{\begin{matrix}\mathrm{Ra} : \mathrm{Fah} : \mathrm{La} : \mathrm{Te} & \\ & \mathrm{Te} : \mathrm{Re} : \mathrm{Fe} : \mathrm{Lah}\end{matrix}\right\}$$

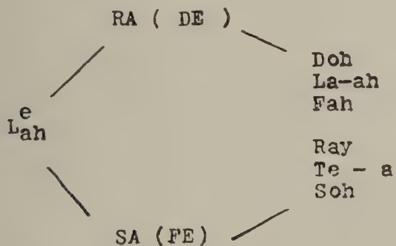
The Core-Envelope Aspect (Tertriadal Matrix)

F Species (" Even " type):—



The axiality of the Pythagorean system is seen to be replaced by the " parasyntonic contiguity " in the Antinominant.

Similarly the " Odd " type:—



Comparative groupings may be found in the C Species.

The formation of mixed Envelopes of each pole is possible. The process gives rise to an optional duality, which behaves as an Axis; although the tones in question (L : πLT) (πTT : T) are a semitone apart.

The resultant chords are of pentad form, but are usually found in either one or the other of their tetrads.

Laxator Mixed Pentad

		F	C
Axis	LT = P	Doh	Me
	LD or ◗	Lah–La	Soh–Se
Quasi-axis	L or πLT	Fah, Fe	Te, Ta
	πLD or ◗	Re–Ray	La–Le
π pole	πL	Te	Fah

Tensor Mixed Pentad

		F	C
Axis	TT	Ray	Ray
	TD or ◗	Te–Ta	Fah–Fe
Quasi-axis	πTT or T	Se, Soh	La, Lah
	πTD or ◗	Mee–Me	Da–Doh
π pole	πT	De	Ma

The experiential occurrence of Antinominant relationships will be discussed later, but it may be noted that the Fundamental " Matrical " forms are not uncommon.

Possibly the utilisation of such forms has been restricted by the fact that both notation and instrumental claviature have been devised for the facile presentation of Pythagorean (hemicyclic) configurations; thus rendering the appearance and technique of antinominantal progressions somewhat complex.

The way in which older musicians clung to Just Intonation is also partly responsible for the state of affairs; which represents a kind of survival of the fittest (? Pythagorean) on present lines.

Musicians are, however, becoming rapidly accustomed to antinominant translation and its consequent developments, and there is little need to preach progress in this direction.

The coincidental forms are met with to a lesser extent although symmetric modifications are fairly common.

The Quinomial polar extremes of both Antinominant and Pythagorean forms are necessarily (as odd-numbered) identical: the meeting-place of both forms is to be found in translation of a tetragonal phase, *i.e.* orthogonal.

In practical terms, the near Pythagorean translations involve the addition or removal of one sharp or flat to the key signature; the corresponding antinominant progressions involve five, so that progressions involving three steps are on the mutual boundary.

There is a method of notation and claviature that has at various times been suggested, and, indeed, backed by a considerable literature.

This consists in forming a scale of oscillants, and regarding the antinominant steps found in the ordinary scales as places where a step up, or down, to a parallel scale occurs.

Taking the natural scale, from Doh to Me, we are in one " region," then we step into the " super-antinominantal " region, and continue until we step out again from Te to Doh.

The reasons why this ingenious system did not " take on " are

fairly obvious; it was nearly, but not quite, as good as the present system, and there is not room for two.

We simply mention it now, to show how antinominantal relations apply to everyday practice, and how light is shed on some of the harmonic practices of the present day by viewing them in the light of Antinominantal relationships.

So far, a certain amount of correspondence has been observed to hold between the Pythagorean and Antinominant forms, which, however, does not apparently lend itself to further extension.

It is, therefore, convenient to turn to another aspect in which the relationship between the two forms is more clearly exhibited, and the applicability in practice defined.

Instead of taking Triads (P : $\genfrac{}{}{0pt}{}{\text{D}}{\text{◗}}$: T) as "nomials," the Tetrads (P : $\genfrac{}{}{0pt}{}{\text{D}}{\text{◗}}$: T : $\genfrac{}{}{0pt}{}{\text{ᗡ}}{\text{◖}}$) may be so employed.

Upon diametrically translating such a Tetrad, it is seen that, since the interval of translation W/2 is that holding, in E.T., between the Determinator and Contra-determinator (being, as semi-octave, achromatically the same whether upward or downward in pitch), the operation merely transposes these second-order components in relative nomination.

The operations may be illustrated, in Solfa terms, with regard to the Tetrads of the Pythagorean Trinomial, in both Species:—

Fundamental Species

Tetrads on the:—

	Laxator.		Prime.		Tensor.	
	Pyth.	Anti.	Pyth.	Anti.	Pyth.	Anti.
ᗡ	Ma or Ray	Lah or La	Ta or Lah	Me or Re	Fah or Me	Te or Ta
T	Doh	Fe	Soh	De	Ray	La
D	Lah or La	Ma or Ray	Me or Ma	Ta or Lah	Te or Ta	Fah or Me
P	Fah	Te	Doh	Fe	Soh	Ra

Coincidental Species

Tetrads on the:—

	Tensor.		Prime.		Laxator.	
	Pyth.	Anti.	Pyth.	Anti.	Pyth.	Anti.
P	Lah	Re	Me	Ta	Te	Fah
D	Fah or Fe	Te or Doh	Doh or De	Sa or Soh	Soh or Se	Ra or Ray
T	Ray	Se	Lah	Ma	Me	Ta
ᗡ	Te or Doh	Fah or Fe	Sa or Soh	Doh or De	Ra or Ray	Soh or Se

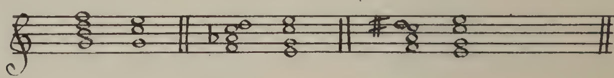
The notation of terms, in Staff or Solfa, is a little inclined to obscurity, owing to the range of tones; but all values are to be understood as E.T. irrespective of practical methods of utilisation.

The rationale of the symmetry of the Pythagorean and Antinominant forms rests upon the analogous contrast between Core and Envelope, and the formulation of the Tonal Trinomial by serial connection, and the parasyntonic proximity, respectively.

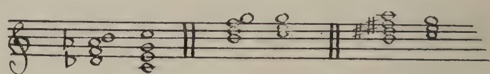
The two forms, so opposed in notational appearance, are coupled up when the determinators of the one, and the contradeterminators of the other, can enter; since these components are both second-order elements, and thus distinguished from Primes, Tensors, and Laxators.

This coupling up is effected by means of the chords known as "Augmented Sixths" (derived from the Tri-heptad and Quintriad Aggregations) in the one form, and radical tetrads in the other, while the symmetrical tetrad is unchanged in constitution by being transposed in form, merely suffering rearrangement.

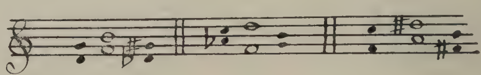
Pythagorean Form.—Radical Tetrads progressing to a Centron Triad.



Antinominant Form.—The Radical Tetrads, translated over a semi-octave, progressing to the same Centron Triad.



The axial dyad, in the above Tetrads, should be noted, viz.—



A "German" Augmented Sixth on La transforms into a "Supertonic Seventh"; and on Ra to a Dominant Seventh.

"French" Augmented Sixths retain their constitution, but are rearranged, by diametrical transformation.

The "Augmented Sixth" chord upon Fe does not often appear in practice. It involves the non-polar diametrical triad of the Centron.

SUBSECTION 5

ABSOLUTE VERTICATION

As premised in an earlier portion of this work, the ineliminable abstract conditions, known generally as " Absolute Vertication," have been so far neglected in order to enable the converse symmetry of the two " species " to be examined in parallel.

It is now convenient to examine these relegated conditions themselves in the abstract, and to re-apply them to the principles of determinate tonality already arrived at.

It has already been mentioned that the Harmonic Series has a real acoustical predominance over all other series of possible tones.

This arises from the physical conditions which make the mathematical theory of Fourier evident: particularly the fact that bodies suitable for the generation of useful musical sounds are selectively homogeneous; and regularity in shape is generally imposed by the condition of linear control over the pitches of sounds required.

The survival principle pointed out by Rayleigh has an important application when the physiological aspect of acoustics is regarded.

Whatever the auditory mechanism may be, it certainly is not symmetrical; and there is every reason to believe that were the unit perceptions concomitant with any but cyclon vibrations, serious modification of effect would result: and in consequence, the freedom and generality now enjoyed in the practice of music (as regards range and class of sound, etc.) would be very much restricted.

Lastly, in the psychological aspect, as investigated by Stumpf and others; the synergy of presentations in the (early) series intervals constitutes a real blending or fusion, whereby it becomes possible to abstract those intervals, and utilise them musically as elements, not only in their original serial position, but even in the inverse form of the Coincidental grouping.

The persistent recognition of Absolute Vertication (whereby CT always appears as FP of the permuted triad) is a definite fact of experience.

The idea promulgated by Rameau as to an imaginary prime

radical (which he termed a "fundamental bass") cannot be maintained; but it is seen that every unit interval in a chord is judged as vertical; and of the intervals in a triad or compound chord, those known as "radicals" (from the bass) have a distinct prominence.

This latter fact is illustrated by the:—

(1) Empirical principles of Thorough-Bass.
(2) The "Faux-bourdon"; in which a series of "consecutive R" intervals is consonant.
(3) The most concordant position of discords (*e.g.* Y+V), the licensed discord of the strict contrapuntist.

But there is one point of physiological acoustics, somewhat disputable, which throws distinct predominance upon the vertication arrangement.

This is the effect discussed by A. M. Mayer, and confirmed by Rayleigh, viz. the fact that a bass sound tends to blot out perception of tones of higher pitch.

Although far from being accepted as a general theorem, this phenomenon is undoubtedly observable.

It has been attributed to a focussing or adapting action of the auditory muscles controlling the tympanic membrane, which are considered to tune the ear to the tone of lowest pitch.

Leaving these disputable points, there is no doubt as to the existence and effect of Absolute Vertication.

The Fundamental Prime is the absolute terminator of its monad series; whereas the same can be said of no other tone.

Application of the criterion of Absolute Vertication leads to the recognition of achromatic position, the Zero-chrome components of chords, etc.; and a discrimination therefore primarily between:—

(1) The achromatic "inversions" of chords.
(2) The distribution and arrangement of tones in chords, apart from the basal component.

In the simultaneous aspect, translation, and progression generally, only appear as passive relationships between chord forms.

The operative view involves consideration of successive factors, conveniently dealt with later.

It may now be pertinent to introduce consideration of absolute Vertication, and to note how the bias affects the Fundamental

Species, with its Order partials (dextral polarity, and educt determinality).

It is only in the Fundamental Species that the Seriopolar type of Matrix is effective as a vertication element.

The coherence, in the Coincidental Species, is not perceptible beyond $C\Sigma^1_7$.

The Polar Tetrads (Radical) of the F Species—

$\boxed{T}$ = Soh : Ray : Te : Fah

$\boxed{L}$ = Fah : Doh : Lah : Ma

are the predominant forms of the type in experience, and when compounded into an Envelope, the determinant identity of the tones Fah and Ma as Polar Contra-determinators is only evident (except under rare circumstances) when the tone Ma is higher in pitch than Fah.

The fact that, acoustically, the Seriopolar aspect of the Matrix is only effective in the Fundamental Species, and that the approximation of the Tensor Heptad to the Hemicyclic type is nearer than the Laxator Heptad, is evidently responsible for the recognition of a distinct chiral bias in progression denoted at an early date by the word "Authentic."

The effect of a tendency to modulate "towards the Subdominant" is well known to all acquainted with harmonic procedure.

The Laxator Series approximates but roughly to the Hemicycle (the interval L(7 : 13) having to represent an Octave), but is much nearer to a Tertriad with permuted prime and tensor determinators—

L : LD : P : P◗ : T : T◗ : TT

which is the chiral reverse of the symmetric form.

The fact that this type of matrix strongly suggests a translation of Locus to the Bi-laxator, is responsible for its rare use; and this also shows that the laeval complement of the Seriopolar Matrix contributes but little to the formulation of the Septomial as a whole, although effective in coupling individual pairs of members.

When the Core triad, which represents maximum stability, is achromatically extended so that the chrome R (between T and WP) is included, certain effects due to the latter are perceived.

R is a peculiar interval, of dual aspect in Degree, being both a Chrome as well as the leading fluent W/B of the Series.

As a component of the infinitely extended achromatic triad, it is elementary and indivisible, which justifies its symbolisation by a primary colour.

But R is sufficiently alike to B in dimensions, to suggest, at its first appearance in the series, the second (bisected) aspect.

When B is heard, the suggestion of a determined triad is prominent, and the same effect in the case of R gives rise to the two "pseudo-chromes" of ultra-second order.

The Contra-triad, as it may be termed, formed by P(6 : 7 : 8) possesses an appreciable acoustic audentity, which may yet become a particular factor in tonal practice.

The interval R can be bisected in two partial species, appearing thus as a composite chrome, viz.—

T ᗡ WP approximated by	Soh	: Ta	: Doh	(F)
	Lah	: Fe	: Me	(C)
T ◖ WP ,,	Soh	: Lah	: Doh	(F)
	Lah	: Soh	: Me	(C)

But, as already known, the recognition of the "bisectors" as contra-determinators, definitely nominates the chord as part of a Tetrad, *i.e.* an Envelope, as contrasted with a Triad, and progression is directed thereby towards the Polar triads, as may be seen from the quotation:—

Pythagorean Illustration

Fundamental Species.—(1) Core to Laeval Polar Triad.

Doh			P		
	Ta			ᗡ	
		Lah			LD (or L◗)
Soh	Soh		T	T	
		Fah			L
Me	Me		D	D	
Doh	Doh	Doh	P	P	P

(2) Core to Dextral Polar Triad.

		Te			TD
	Lah			◖	
Soh	Soh	Soh	T	T	T
Me	Me		D	D	
		Ray			TT
Doh	Doh		P	P	
		Te			TD

Coincidental Species.—(1) Core to Laeval Polar Triad.

Me	Me	Me	P	P	P
Doh	Doh	Te	D	D	L
Lah	Lah	Soh	T	T	LD (or L◗)
Me	Fe		P	ᗡ	

(2) Core to Dextral Polar Triad.

		Fah			TD
Me	Me	Ray	P	P	TT
Doh	Doh		D	D	
Lah	Lah	Lah	T	T	T
	Soh	Fah		◖	TD

Antinominant Illustration

Fundamental Species.—(1) Core to Diametrical Dextral Polar Triad.

Doh	Ta		P	ᗡ		
Soh	Soh	La	T	T		TT
Me	Me	Fah	D	D	π	TD
Doh	Doh	Ra	P	P		T

(2) Core to Diametrical Laeval Polar Triad.

	Lah	Te		◖		L
Soh	Soh		T	T		
Me	Me	Fe	D	D	π	LT
Doh	Doh	Re	P	P		LD
		Te				L

Coincidental Species.—(1) Core to Diametrical Dextral Polar Triad.

Me	Me	Re	P	P		T
Doh	Doh	Te	D	D	π	TD
Lah	Lah	Se	T	T		TT
Me	Fe		P	ᗡ		

(2) Core to Diametrical Laeval Polar Triad.

Me	Me	Fah	P	P		L
Doh	Doh	Ra	D	D		LD
Lah	Lah	Ta	T	T	π	LT
	Soh	Fah		◖		L

The permutability of the Determinators and Contra-determinators adds further to the chord forms in this process of directional progression.

By the gradation of audentity according to "Bundling" analysis, it is seen that a given chrome is predominant when it occurs between the Bass tone and an upper member. It is less evident when between two upper members of a chord.

R is therefore brought into prominence by the achromatic arrangement known as "Second Inversion," *i.e.* the F Tensor, or C Prime, being in the Bass.

If B is in this position, its fission into either V : G, or G : V, introduces no new member into the triad, or chord of which the triad forms part.

If a second-order chrome G or V is in this position, it is subject to no analytic influence of any importance.

But if R is prominently brought out, in such a case the potential or suggested contra-determinator tends to obtrude itself, altering the character of the triad from Core to Envelope; therefore the Second Inversion of a Triad loses somewhat of its coherent unity, appearing as a latent discord or compound chord, and emphasising the aspect of R as a fluent of W/B type.

Thus it appears that there are certain agencies which tend to suggest or direct progression along definite Pythagorean or Antinominant channels.

Similarly, as regards Commutation, the Coincidental Prime Triad may appear as either a compound "semi-nomial" chord, *i.e.* a state of cyclic transition, or as a recessive quasi-triad of the Seriopolar (Tensor) Series, viz. T(9 : 11 : 13), and thus as part of the Fundamental Envelope compounded with a part of the Core.

The Coincidental Prime Triad may also appear as a Secondary Envelope about the dyad TD : TT, which is part of the tensor triad, and as such, part of the Primary Envelope.

Hence, Vertication introduces a distinct directive bias of progression towards a Fundamental Triad.

Further considerations of this type would lead to many examples of directive agencies, not necessarily very potent, which concern progression, and with which the tone artist works.

However, by these steps, we are led. to the threshold of

Successive Tonality, where the whole subject is treated from a different generative basis.

It may not be out of place to refer to the usual method of harmonising Modal Melodies, as evidenced particularly in the cadences, where it will be seen First-order Tones act as "Leading Notes."

The facts of history tell us that Second-order chromes had not the recognised position they now possess and consequently the chordal schema of that time was on an altogether more limited basis, being limited to First- and Zero-order Chromes, the Second-order elements appearing as neutral members with little or no influence upon progression.

Hence the neutrality of species, from which it is possible to select any arbitrary tone of the "natural" scale as Tonic, and the whole six triads could be used indifferently and interchangeably, without any (or very little) discrimination into redominant and recessive.

The only limiting factor was the Tritone, which terminated the chain of triads on either hand. In order to supervene this barrier in practice various devices were introduced, such as the B-rotundum : B-Quadratum, and the Mixo-scales, etc.

The weakness of "determinator" chordance conferred upon the "Modes" the freedom they apparently possess, because the occurrence of the mediants in either species did not affect the chordal progression, but only served to distinguish the "locus" of one mode from another.

The effect of modal progressions to the modern ear has a curious "inside-out" character, due to the fact that progressions between predominant and recessive triads which are possible, but subsidiary, in the Ter-triadal aspect, become the essential elements, while the Trinomial of triads sinks to a more subsidiary position in the presentation.

Some of this is undoubtedly due to the extraneous circumstance that the Ionian Mode was studiously avoided by the ecclesiastical revivers of quasi-Greek Modes. Whether this avoidance was due to a pure æsthetic feeling, or, as more probable, to a desire to set apart the more austere elements in definite contrast to secular methods (see Pope John's fulmination), is not easy to determine.

Also, when considering the historical evidence of tonal methods, it must always be remembered that by far the greater amount of

notational data comes from the somewhat specialised branch of musical art in which its practice was associated with people who could read and write; while the popular and secular music of those times was mostly transmitted orally, the " folk-songs " of to-day having doubtless undergone countless modifications since their first performance.

The effect of the secular upon the ecclesiastical " authorised " music was both positive and negative. There are many evidences of surreptitious borrowing, as well as studious avoidance (the same process may be observed going on at present between so-called popular and classical music), but practically only one party in the process had pens and ink.

There is plenty of scattered evidence that the mediæval ear did recognise the " determinal " character of Second-order chromes, *vide* the " Gimel," Glarean's remarks, and the curious " Sumer is i-cumen in."

But recognition and adoption are not always identical, and a good deal of accretion had to be unlearned before Chordance, as we know it, came to its own.

SUBSECTION 6

COMPOSITE SPECIES

Attention may now be directed to the Composite Species, and particularly to the predominating matrical types.

A distinction may be drawn between a scalar mode and the constitution of the scalar matrix, out of which the mode is determined by arbitrary location of limits.

This distinction is somewhat loosely observed in empirical theory. It is usual to talk of the present Major and Minor Modes as if they were comparable with their Greek and Ecclesiastical synonyms.

This may be correct when considering the Fundamental as Major, and the Coincidental as Æolian or Declinear Melodic Minor " Modes " of the same natural or hemicyclic matrix, but it is obvious that both of these forms are as composite in constitution as the Harmonic Minor, or Aclinear Melodic Minor Modes, when viewed from a Tertriadal, *i.e.* chordal, aspect.

The composite nature is inherent in the Trinomial common to

all forms. Viewed in the light of First-Order Species, the group is composed of:—$T^{-1} : P : T^{+1}$.

When the determinators are all fundamental, we have the Major Mode, whose members are identical with those of the Relative Minor (Descending Melodic Form).

With the determinators all permuted to coincidental, we obtain the Tonic Minor (Declinear Melodic Form).

Converse conditions hold with a Coincidental Trinomial.

In either of the "Natural" scales, the determinator of the Laxator triad is opposed to the species of that triad; whereas in the Harmonic Minor Scale, the determinator of the Tensor triad is permuted (as regards notation) to the species of its triad.

Both Modes are therefore composite, and it is only the appearance in notation, and on the keyboard, that suggests an artificial character in the Harmonic Minor Mode.

(The performance of the V leap presents little difficulty when the sense of terminal dissimilarity has been overcome.)

It is probable that the heterogeneous appearance may have had somewhat to do with the translatory treatment of the mode; at any rate, it tends to invite an escape from the apparent rigidity of the "natural" scale.

The outstanding feature of the harmonic minor mode is the difference of species between the determinators of L and T whereby the interval (Super-oscillant) becomes in E.T. equivalent to the chrome V (Y).

The variants of the Minor Mode known as Melodic are obtained by permuting the Enveloping Tetrad in the direction of progression.

Considering the scale as made up of two "Semi-scales" (Greek Tetrads) respectively Projective and Retractive, it is seen that the "Retractive" semi-scale is fluid as to its internal character, in contradistinction to the fixity of the Projective.

The Prime Determinator is normally coincidental, since it nominates the species in distinction to the Major (Fundamental) Mode. When it is permuted, we have the "Picardian" or "Hungarian" variety.

This optional permutability or "flotation" of the second-order components has an important bearing upon a very interesting question.

It must have occurred to the reader to ask how it was that the

Modal system was developed to such an extent, and persisted over such a long period of musical history, when the natural acoustical conditions of Tonality would seem to favour the Tertriadal Modes, Fundamental Hemicyclic and Symmetrical.

This argument is still more weighty when Cadential progression is considered; and the whole question is favoured by notational practice.

If we grant that the perceptive determinance of interval diminishes in grade inversely to their order (there are four second-, two first-, and one zero-order chromes), and that the melodic interest originally predominated over the justly intoned (owing largely to the facility of attainment of the former), then we may be prepared to admit that the ears of our ancestors were not so finely attuned to second- as to first- and zero-order chromality.

Consequently, the casuistic "flotability" of the second-order tones was a very real factor in early chordal procedure, while the Pythagorean Quadrant was the sheet anchor.

It would appear that the neglect of second-order just intonation may be partly attributed to the Pythagorean method of deriving second-order relationship, which is based upon sound melodic practice, and the recognition of first-order predominance.

The modes, as far as can be ascertained by the study of ancient works, appear to have been derived in two ways:—

(1) By the Relative transposition of the terminals of scale along the general "natural" or diatonic range.
(2) By the Tonic modulation of intervals within two fixed octave limits.

In the following Table, the First- and Pseudo-first-order intervals of the "Modes" are given, and their "Perfection" is estimated thereupon. The Second-order tones are regarded as outside the problem; but, of course, it must not be implied that this was the determinate and conscious procedure of any age or school of practical or theoretical musicians.

It is to be noted that the Modes Nos. 1, 7, and 9 are dually perfect. Those numbered 13 and 3 are singly perfect; while those numbered 5 and 11 are dually imperfect.

If we neglect the interval (T : J) and consider only the Trinomial, No. 5 is more perfect than No. 11.

The flat supertonic of the Third Mode could be read as a

MODAL TABLE

Greek Name.	Ecclesiastical (Authentic).	No.	Terminal.	F Aclinear.							C Declinear.						
				L		P		T		J	L		P		T		J
Lydian .	Ionian . .	13	Doh	Fah	B	Doh	B	Soh	B	Ray	Soh	B	Doh	B	Fah	W/2	Te
Phrygian .	Dorian . .	1	Ray	Soh	B	Ray	B	Lah	B	Me	Lah	B	Ray	B	Soh	B	Doh
Dorian . .	Phrygian .	3	Me	Lah	B	Me	B	Te	W/2	Fah	Te	B	Me	B	Lah	B	Ray
Hypolydian .	Lydian. .	5	Fah	Te	W/2	Fah	B	Doh	B	Soh	Doh	B	Fah	W/2	Te	B	Me
Hypophrygian (Ionian) .	Mixolydian .	7	Soh	Doh	B	Soh	B	Ray	B	Lah	Ray	B	Soh	B	Doh	B	Fah
Hypodorian (Æolian or Locrian) .	Æolian . .	9	Lah	Ray	B	Lah	B	Me	B	Te	Me	B	Lah	B	Ray	B	Soh
Mixolydian .	not recognised	11	Te	Me	B	Te	W/2	Fah	B	Doh	Fah	W/2	Te	B	Me	B	Lah

permuted bilaxator determinator: valid in the Tri-heptad, Quintriad, or Nonomial.

Theory thus concurs fairly well with experience, but it is not easy, when studying old modes, to be sure where the artificial element supplants the natural expression.

It is to be noted that the First Mode (Ray) is the Centre of Species Symmetry; the mirror of Von Oettingen.

In the abstract consideration of Tonality, it is seen that the "Major and Minor Modes" represent the predominating forms of a large number of possible types.

These subsidiary types are frequently used in musical manifestation, both in chordal and scalar form; and give rise to a notational appearance of complexity, especially when the triads are more latent than actual in constitution.

The ear, however, is not governed by the exigencies of notation and instrumental control, and judges by the principles of determinate tonality and structural relevancy.

Hence, in some of the older treatises, certain configurations, which the ear approved, but which did not obviously fit into the particular scheme of empiric theory, were termed "licences," to the great bewilderment of the student, and to the disparagement of "theory" in the eyes of many practical musicians.

It must be generally admitted that nature does not usually grant "licences"; and the student may be well advised to follow the method of science in regard to any apparent anomalies.

From a careful study of historical evidence, Helmholtz came to the conclusion that the harmonic minor mode resulted from a fusion of the (Ecclesiastical) Dorian, Phrygian, and Æolian Modes.

Viewing this in the light of the theory just enunciated it appears that the evolution is the direct evidence of some hyperacoustic factor determining the second-order tones.

Evolution is always conditioned by the possibility of a final result; and is evidence of an operative set of factors.

If the Harmonic Minor Mode had been indeterminate in tonality, the fusion might have indeed been effected by tone-experimentalists, but the type would never have persisted and gained the predominant position it now holds.

The æsthetic effect of minor chords and configurations has been fully discussed by various authorities without any very convincing conclusions—which is not unusual in æsthetics.

Possibly, the multiplicity of roots in the minor triad has something to do with the effect of solemn firmness that characterises (slow) presentations.

The rationale of the "Tonic" Minor depends upon the interchangeability with the Major, due to the axial quadrinomial.

This enables the two species to be freely worked together on the same basis of vertication, and same first-order framework, without precluding the chordal relationships and progressions peculiar to the purely coincidental aspect, as in Oettingen's Phonic System.

The rationale of the "Relative" Minor appears in the concomitant recessive aspect with respect to a nominate Major (and *vice versa*).

This is due to the potentiality of the quadrinomial afforded by the three determinators of similar species, plus the mutual Yoke.

In practice, it is found that the predominant and recessive species are often temporarily reversed: the one species waxing with the waning of the other; somewhat like the pictures of the old dissolving-view magic lantern.

The fact that a Minor Mode has both a Relative and a Tonic Major, mutually orthogonal to the Nominant, should never be overlooked. Some theorists appear to imagine there is a heaven-sent rule of monogamy about the matter, and argue violently as to which is the legal mate of the nominant.

A little of the energy diverted from controversy to research would be a good thing. Still, evidence often results from strife of this kind.

CHAPTER XI

SUBSECTION 1

PROGRESSION

We now arrive at the progressional division of Tonality which, strictly speaking, does not come within the purview of the Simultaneous aspect.

The necessary implication of questions involving time, makes it convenient to relegate the subject to the Successive aspect.

But in order to obtain a comprehensive idea of Tonality, it is perhaps allowable to travel a little outside the logical scope of the inquiry; and by stepping over some of the abstract limits imposed for convenience of study, we may anticipate matters considered in a later section: thus, by retrospection, facilitating comprehension of matters already dealt with.

This division must, of necessity, be restricted within a brief space; the idea being to exhibit the " static " or passive relationships in their " active " or operational form.

The relationships of the elements and groups contained within a tonal system are conveniently considered in the two abstract aspects, nominantal and operational.

Nominantal relationship is a purely passive expression of how terms stand to one another, by which they are named; irrespective of any idea of progression.

The Operational aspect regards the passage from one note, chord, matrix, etc., to another, as expressed by the effect of some definite action.

Actually, the progression is nothing more than the discontinuance of one set of sounds, and the initiation of another, with or without intervening silences.

The physical and physiological effect of sound succession at the rates ordinarily in use is of little moment; but the psychological aspect is that in which the idea of a progression of sounds appears as the motion of a continuous entity.

The term " Motion " may therefore be applied under reserve to describe the succession of tones, chromes, chords, etc., in determinate tonality.

The Tone may be regarded as a point in the pitch space which is drawn out into a line in Time. This " Line-entity " may be known as a Phonon.

Its characteristic identity is the important factor to notice.

The idea of Progression therefore involves consideration of Protension, *i.e.* extension in time, in the same way that tonality connotes extension in the " space " of pitch.

The term " protension " is used because the idea of duration itself is of different order; musical " tempo " being elastic with respect to chronometric projection.

When the progression of a tone involves alteration of pitch, such a tone is said to Move.

Motion is distinguished into aclinear and declinear, according to the direction, upwards and downwards.

Motion may be Chromal (Arpeggial), Fluental (Scalar), or may, as the liminal motion known as enharmonic progression, vanish in E.T.

The tones from and towards which a definite tone progresses may be known respectively as its (proximate) precursor and successor.

When a tone does not " move " in pitch, but alters in nominantal relationship towards a system of reference, such a tone is said to Change, over a chromal, fluental, or liminal range.

The element of a group, which remains unmoved during a progression, constitutes the Axis of that progression, about which the free members turn.

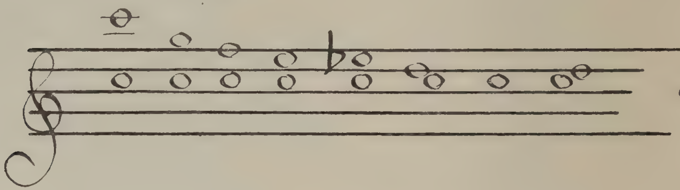
Axes may be tones, chromes, triads, etc. The progression is defined from the number of fixed elements, as uniaxial, binaxial, triaxial, etc.

The important Commute axis is the Yoke (and its orthogon).

We have already seen, that to compare the respective Series Chromes, in magnitude, it is convenient to bring them to a common starting-point; *i.e.* to reduce the frequency so that one of the terminal tones of each chrome coincide in pitch. (Least common multiple method.)

This is equivalent to the transposition of the Series Chromes

from their respective positions to a common axis as illustrated in Staff notation by:—



&c

The allocation of the axis enables the other free tone to be expressed as a fluent, " leaping " or " stepping," which converts one chrome into another. Thus the idea of axiality is seen to be involved in the consideration of Tonal relationships of more than one Degree.

The principle can be broadened by reversing the aspects of Space (Pitch) and Time.

A note appears as a Locus or point, on as many independent co-existent (orthogonal) linear dimensions as its characteristics can vary in.

It can vary in Duration. Therefore an axis appears as the " protension " or " drawing out " of an identity in time, between two (or more) comparable simultaneous nominate states.

This is quasi-graphically illustrated in notation by the arbitrary method of writing time horizontally.

A note can vary in pitch. Such a " Line-entity " protended in time and extended in pitch constitutes a Flexion, distinguished as a Clinear (Ac or Dec) Phonon.

The conventional method of representation in Staff notation is upward or downward.

In music, we are not concerned with pure Flexions (except as portamentos or glides—a rarely-to-be-used ornament), but we are concerned with the nearest clinear representative in the form of the Scale, which thus stands for a connective agency in pitch, analogous to the axis in duration.

In a similar manner, a tone can be " Intensively " drawn out along the continuous and independent variable of " Loudness " from vanishing to overpowering limits.

In Tone-tint, there are a myriad independent lines of alteration which can be taken between the pure Cyclon and the limit of evanescence into noise.

These latter variables have, so far, no definite notation in practice; although the records of mechanical players partly show them in their own peculiar notation.

Consideration upon these secondary points must be deferred for the present.

Meanwhile, it is convenient to consider Chordal relationship in its operational aspect, Progression, from the standpoint of connection in time by the Axis, and in pitch by the Scale.

The principle is capable of extension. Thus, axiality can be operative with no actual axis, and " clinearity " is evident in the undulating " change-note," the sequence, etc.

The basis of the whole system inheres in the general principle of hyperacoustic presentation, viz. the psychological universality of manifestation; whereby all the analogues of perception and conception, of personal intra- and extra-spection are paralleled.

This principle, styled briefly that of " Universal Presentation," will be discussed later.

SUBSECTION 2

PROGRESSION FROM AN AXIAL ASPECT

The relationship between two chords can be conveniently expressed in two sets of terms:—

(1) By considering the motion of the non-axial components.
(2) By regarding the change of the Axis (if any).

In the present subsection, the second method is examined, although both are really inseparable in the general consideration.

When two serial chromes have to be considered together, they are transposed so that one tone of each is axial.

The progression can then be expressed by means of an Axial Coefficient in the form of a fraction; the denominator being the original nomination of the Axis, and the numerator representing the new value.

By inverting this operator expression, the reversal of the progression is indicated.

The primary indication concerns Species, and its " partial " statement.

The operational symbols suitable to denote species follow from

the methods already discussed under the heading of Total and Partial Specification.

Thus f denotes a fundamental, and c a coincidental species totality, which may be restricted to any particular order by the application of indexes.

f^0	corresponds to	positive or upward Achrome direction
c^0	,,	negative or downward ,, ,,
f^1	,,	dextral polar direction
c^1	,,	laeval polar ,,
f^2	,,	" educt " or co-zeral determinality
c^2	,,	" adduct " or anti-zeral ,, *i.e.* to permutation

The " original " species of a term is indicated by the denomination in reciprocal values $^1/_f,\ ^1/_c,$ or by a negative sign to the index $f^{-n},\ c^{-n}$.

Commutation is expressed by terms:—$^c/_f,\ ^f/_c,\ c^n f^{-n},\ f^n c^{-n}$.

Neutral or ambi-specific expressions do not contain any of the above symbols, but general commutation might be expressed by the arbitrary symbol of division $\div$.

The expression of Order may take the form of:—

$$\frac{N^a}{N^b} \text{ or } N^{a-b} \text{ on the same basis.}$$

Phase, representing a relationship tending towards recurrence, is conveniently expressed by a numerical fraction, whose numerator indicates the number of times a unit relationship is taken, while the denominator denotes the complete period in the cycle of such units.

The axes of basic type may now be considered.

In a coherent group of tones, all the members are " axial." Taking a whole series, the intensity of any arbitrarily selected number of members may be preponderantly increased.

In the present case, only the Triad and its enveloping Tetrad are under consideration; each component implying actual or latent co-axiality with the members of Core or Polar triads.

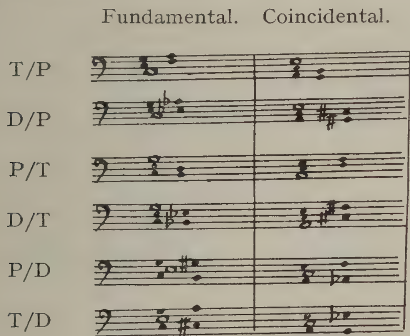
The triadal validity of any tone component is three-fold, and of any pair of tones, two-fold.

The envelope is conveniently dealt with in its aspect as a compound of the pair of polar triads, to each of which the three-fold method applies.

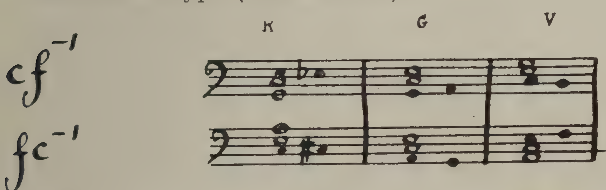
The axial connections of a given triad are consequently $3 \times (3-1)$ uniaxial, and $3 \times (2-1)$ binaxial links. The latter may, for convenience, be also considered as chrome axes.

Uniaxial relationships are independent of Species, but binaxials are commutative, owing to the inherence of a given triadal component in the two kinds of converse species respectively.

Thus the Uniaxial type:—



The Binaxial type (Chrome axial):—



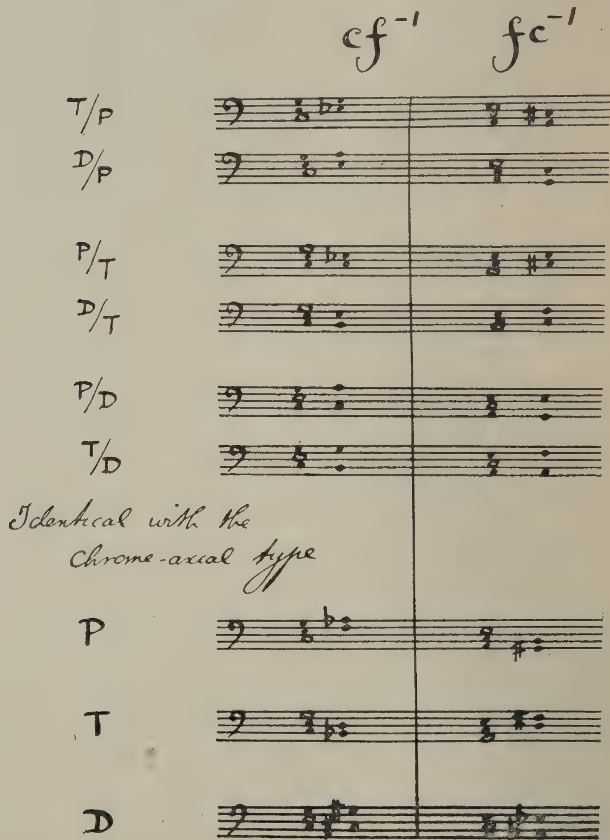
It will be noted that the Chrome-axial type of first order corresponds to permutation; also that the axes of type G and M are reversible in the concomitant species.

The whole of the axial relationships are shown over page.

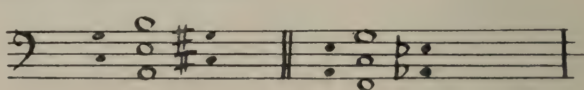
The last of these cases cf^{-1}D, fc^{-1}D is, owing to the paraphonic motion of the first-order chrome, a somewhat dissonant progression, infrequently met with in experience.

The recessive audentity of the moving components is responsible for the acceptance of this progression as part of the compound di-triad, or envelope of second grade, in which is seen the simultaneous permutation of the two determinators in parallel: any given case being really in its concomitant species.

The "first order" of the interval formed by the two moving tones conflicts with the "second order" implied in their per-



mutation. Consequently, the effect is indeterminate in spite of its apparent rational axiality.



Turning now to the simple trinomial expression of relationship:—

Pythagorean	$L : P : T$
Antinominantal odd type . .	$L : \pi P : T$
Antinominantal even type .	$\pi L : P : \pi T$
Diametrical Pythagorean . .	$\pi L : \pi P : \pi T$

The Core contains two bivalent members, the Axes P and T, which unite the polar components in the Envelope.

The third member is univalent as it is not an Axis. Regarded as dual, it appears as the infra-second-order relation of the Prime, and the corresponding ultra to the Tensor, and so " splits " into two aspects.

(The idea may be put into the form that the Tonic Mediant is Submediant of the Dominant and Leading Note of the Subdominant, its corresponding inclinations being converse.)

The Quinomial form is seen to possess two subsidiary axes in the Laxator and Tensor, uniting the Secondary Envelopes with the Centron.

$$LL : {}^{\pi}_{Z}L : P : {}^{\pi}_{Z}T : TT$$

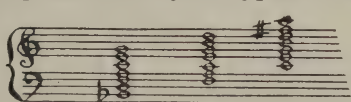
The trinomial relationship is concerned with the Septomial Matrix, and its Heptadial extension is seen to be of a different experiential type, which verges on the Translational.

In a particular aspect, the recessive concomitant triads of the Septomial appear as compounds of a transitional type of envelope.

Similarly, in the Tri-heptad, two out of these three chords appear identical (and therefore axial) with two of the three local polar recessives.

Upon viewing the Tri-heptad in its Quintriad form, the Axial Triads are easily noted.

Thus in the F Species, hemicyclic type, we have:—



The degree of axial audentity of the Prime triad is three, that of the two polar triads is two.

Similar conditions hold with the extensions to Septriad, Nontriad, etc., and Tetrads may be united uni-, bin- or ter-axially, presenting different grades of " Poles " and envelopes on each side.

In the Symmetrical type, the permutability of the second-order components does not vitiate the axiality, which is of the maximum audentity in the case of First-order elements.

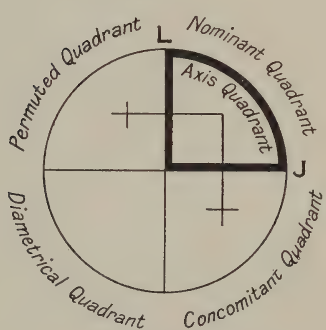
The hemichromal axis W/2 is the uniting element between the envelope chords (Tetradal Form) in which it inheres.

The "tettarto-chrome" W/4 (equals V) is also a factor of the same type.

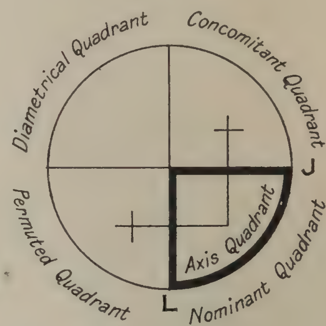
The quadrant of dodecanal phase is the axis uniting two orthogonal hemicycles, and the Trisectant similarly unites the two sesqui-sectants of the Cycle.

The Nominant Quadrant unites the Permuted and Concomitant Hemicycles; the respective tone axes changing in order by one unit of index.

F.



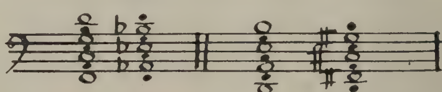
C.



$$\frac{N^2}{N^1}(L : P : T : J) = +\frac{\pi}{2}(L : P : T : J)$$

$$\frac{N^1}{N^2}(L : P : T : J) = -\frac{\pi}{2}(L : P : T : J)$$

These relationships appear in staff notation as:—



where the "open" notes represent precursive and successive "first"-order intervals, which are translated orthogonally in phase.

The transposition change occurs about the diametrical axis (L : TD), which is not found in the triad.

Experience confirms the fact that this is the most unrelated presentation from a triadal point of view, since it definitely involves bi-polarity, *i.e.* and consequently the characteristic of the Envelope.

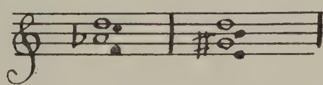
The occurrence in the Tetrad is to be noted:—

(1) Symmetrical Tertriad.

(2) Seriopolar members·
(*a*) Tensor.

F (5 : 7) to C (7 : 5)

(*b*) Laxator.



The "Recessive" Concomitant Tensor triad, in the symmetrical (composite) type of matrix, is a double-violet chord, similar to the Link. The functions of both chords in uniting the poles of the Envelope are evident.

The interval common to both chords is (J : L), the tone J, of course, being common to both species, and thus representing functions in a maximum degree.

The tone J forms first-order chromes with the Tensors of both concomitant species, and also with the respective Laxator Determinators or Tri-tensors.

It forms second-order ultra chromes with the specifically concomitant interchangeables L and TD, and hence becomes the "Centroid" of the Dual Concomitance.

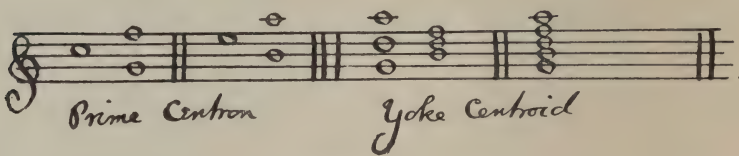
No actual presentation can possess two predominant species at one time, but any aspect of consideration which involves both species indifferently is typically represented as to centroid "locus" by the Yoke.

Hence J stands in comparison with the Prime. This latter unites the converse Poles of the Trinomial by bonds of coherent first-order relationship, from which springs the Tertriadal, Terheptadial, etc., aspects of the Matrix.

J, on the other hand, unites the homonyms of the concomitant

species, standing in the middle of the respective F and C Pentad Envelopes.

$\begin{smallmatrix}F\\C\end{smallmatrix}\Sigma_{I}^{9}\ T$



A similar function inheres in πJ (which, it is noted, bisects the complementary chrome M) as regards the Laevo-nomials of the cycle.

This is seen in considering the derivation relationships of the "Link" pseudo-triad and the "Recessive Tensor Triad," when the Poles are translated to their diametrical equivalents.

The Link becomes:—

$$\pi(\mathrm{TD} : \mathrm{TT} : \mathrm{L}) \text{ equals } \mathrm{L} : \pi\mathrm{J} : \mathrm{TD}$$

The Recessive Tensor triad becomes:—

$$\pi(\mathrm{TT} : \mathrm{L} : \mathrm{LD}) \text{ equals } \pi\mathrm{J} : \mathrm{TD} : \pi\mathrm{TTT}$$

The triad in the last case is an antinominant step below ▽ and a similar step above △ ◗.

The axiality of the Yoke J (and its diametrical) is also shared by the dyads which it symmetrically bisects.

These are the integral multiples of the Oscillant, which, it will be remembered, "oscillate" between con- and dis-cords (in E.T.), viz. G, W/2, M, (W−O), W, (W+O), etc.

The coupling of the Symmetric Envelope (applicable to either species) is seen to be as follows:—

Envelope.	Triad Components.
$\mathrm{FP}^{-1} = \mathrm{(C)P}$ $\mathrm{FPD}^{-1} = \mathrm{L}$◗ $\mathrm{FPT}^{-1} = \mathrm{L}$	Laxator Polar (Coincidental Prime) Element
TT TD T	Tensor Polar Element

The same, in terms of Tetrads—

$FP^{-1}=P$
$PD^{-1}=$ L◗
$PT^{-1}=L \quad =$ Tꓷ
Pꓷ$^{-1}=J \quad = TT$ } Axial Chrome
TD
T

The Axial chrome is seen to be actually the "hypo-violet" T : ꓷ=6 : 7, an ultra-ultra second-order chrome of the Series which is represented in T by the centre violet dyad of the Symmetrical Tetrad.

The hypothetical pitch centroid in this case is the neutral determinator PĐ about which the Envelope is symmetrical. This value cannot, of course, be regarded as a "key-note" since it is not an actual tone of the system, but it is none the less the "centroid" of the symmetrical system.

This is one of the points commended to the notice of De Morgan's "Paradoxers."

The hypothetical centroid is represented by its two nearest tones P◗ and PD, which in the Composite Matrix appear as the "ordinary" and "Picardian" prime determinators (respectively from the F aspect).

A symmetric Heptad may thus be developed. In this group of tones any respective pair is symmetrical about the others and the Neutral Prime Determinator.

The practical value of this schema is in the partial coherence which exists between members, whereby the whole, if not actually coherent, is a consistent abstract system.

The respective intervals are given:—

PĐ				
P	B/2			
L◗	G			
L	V			
TT	V	Y	B or R	(W × 2)
TD	V			
T	G			
PĐ	B/2			

PĐ thus bisects V, Y, B, and 2W.

The two axes connecting a Triad with its Envelope are recog-

nised as those most frequently (but not solely) used as " Pedals " or " Organ points."

This aspect particularly concerns progression, but it reveals the characteristics pertaining to these two tones in representing and retaining the locus of the Matrix throughout the most extreme chordal vagaries of other tones.

Another experience of succession may be noted in this connection, viz. that contrary diatonic motion outwards from the Prime presents dyads terminating in the Poles, followed by a reversal of the chordal succession by retraction to the achrome.

This appears as a " wave " of contrasted Matrical elements, whose successive members also belong to the contrasted groups.

P : J : PD (CP) : L	T : LD : TD : WP
P : TD : LD (CT) : T	L : PD : J : WP

In the Hemicyclic aspect of the Matrix, both in Pythagorean and Antinominant types, the seven notes of the Septomial have no significance as chordal elements.

Each component is of equal chordal audentity, and the equation of series species prevents any simultaneous chordal development.

The arrangement is thus purely of melodic formulation: an aspect of succession.

But it approximately coincides with the Tertriadal (and Seriopolar) aspects, and with them, ineliminably inheres in the Septomial Matrix.

The Tensor Heptad presents some features of interest apart from its obvious Verticate significance, which is due to the closeness of the approximation between its members and the other aspects of the " Hemicyclic " Septomial.

The Radical analysis of its chromes presents the following predominant components:—

Fundamental.		Coincidental.	
T_{13}	quasi Y	T_1	quasi Y
T_{11}	quasi R	T_5	quasi R
T_9	Pyth. just O	T_3	quasi O
T_7	Intercordant (Indigo)	T_7	quasi A
T_3	Just B	T_9	quasi B
T_5	Just G	T_{11}	quasi G
T_1		T_{13}	

The Tensor Heptad falls naturally into two sub-groups of Tetrads (near and far):—

T(1 : 3 : 5 : 7) and T(8 : 9 : 11 : 13)

which are seen to approximate to the two primary types of general Tetrad.

The implication of the E.T. system leads to various "readings" of the Tensor Heptad, of which the foregoing example is one.

The predominance of this subdivision is seen upon considering the Fundamental Prime Heptad in the following sub-groupings:—

P(1 : 3 : 5 : 7)	P : T : D : ᗡ
P(8 : 9 : 11 : 13)	quasi P : J : L : LD

The second of these two chords is conventionally known as the "chord of the added sixth" (2nd Inversion), and, upon deleting J, becomes a Laxator Triad.

Since the approximation in the case of the Tensor Heptad is much closer to the Septomial than with the similar series of the Laxator, it is easy to see on which polar component the bias of Vertication preponderance rests.

The actual experiences of audition as regards hearing the tonal or group-chordal elements in a Series is elusive, and requires perceptive training; but it is generally evident after attention has been specifically drawn to it.

The effect is as if the upper part of the series inclines to a Laeval Chordance in polarity.

This, of course, applies to the Series above any tone, but in the case of the Tensor (and in a lesser degree also with the Laxator) it happens to coincide approximately with the Ter-triadal and Hemicyclic schema of the matrix.

From considerations of analogy, the idea may arise as to the possible existence of a "seminomial envelope."

It is, however, seen that the rationality of the uniaxial polar envelope depends upon the fact that the elements of the polar triads, plus the core, make up the complete septomial matrix. This, of course, does not occur with the "seminomial" envelope.

But the latter may appear as part of the envelope of an envelope, viz. a "conserved" secondary envelope, consisting of two semi-polar triads, which are the concomitant commute prime and laxator triads:

Fundamental Case

Primary Envelope.	Secondary Envelope.	
	Te	
Lah or La	Soh or Se	CL Triad
Fah	Me	
Ray	Doh or De	CP Triad
Te	Lah	

The Tervalence of Tetradial Axiality may be seen in the case of the Envelope about same:—

Fundamental Example

Laeval.			"Core."	Dextral.		
Uniaxial.	Binaxial.	Triaxial.		Triaxial.	Binaxial.	Uniaxial.
Soh						
Me–Ma	Me–Ma					
Doh	Doh	Doh				
Lah–La	Lah–La	Lah–La	Lah–La			
	Fah	Fah	Fah	Fah		
		Ray	Ray	Ray	Ray	
			Te	Te	Te	Te
				Soh	Soh	Soh
					Me–Ma	Me–Ma
						Doh

The triaxial, or proximate, relations on each side will be easily recognised in their various types.

The seven tetrads thus represent the possible variants of regular chords of this type occurring in practice.

The binaxial relationship of the extreme dyad in a Triad is limited to that between the determinator and its permute.

But in the case of the Envelope Tetrad, the binaxiality of the extreme dyad connects the Pythagorean and Antinominant Tetrads, as follows:—

	Fundamental Case.		Coincidental Case.	
Axes	Lah	Lah	Fah	Fah
Fluents	Fah	Fe	Ray	Ra
Fluents	Ray	Re	Te	Ta
Axes	Te	Te	Soh	Soh

The diametrical relationship of the component Triads in the above case is shown by the lines.

The whole progression is one in which the Hemicyclic Tetrad Envelope of a Pythagorean Triad becomes $\pi\Sigma_1^7 L$ and *vice versa.*

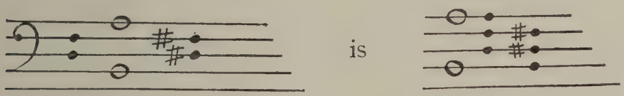
The case is interesting also as one in which (notationally) first-order components appear to be "permuted," which is irrational.

By subdividing the operation into two Triaxial stages, it is seen that the two "fluent" tones are each, in turn, determinators of transitory triads, and as such permutable.

The binaxial oscillants of a Tetrad may however be looked upon in another aspect.

This is the case in which the extremes (Prime and Contra-determinator) change over in name, with the result that the triad portion of the chord swings over.

Fundamental example:—



The operation denotes a commutation and a shift, while the two discordant tones remain axial.

Thus it is seen that the Order of the two internal tones is transposed; so that the "univalent" tensor becomes the new determinator, and as such, permutable (bivalent).

One of the two values of the old determinator becomes the new univalent Tensor.

Whenever a tensor appears to be permuted it is a sign that it has become a determinator or contra-determinator.

In that case, some other element acts as axis; in the illustrated case it is the oscillant dyad, whose audentity is considerable in the Envelope.

(A pair of extreme spaces /E and E/ act thus as axes.)

The basic principle involved in progression is the maintenance of Locus and sub-loci.

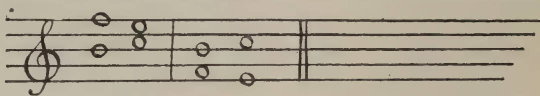
This is effected by the persistent distinction between a Core or Centron (which is the centroid of an equated group) and its Envelopes (which are equated).

Progressions of the type Envelope-Core and *vice versa* may be defined as Acentric and Apocentric.

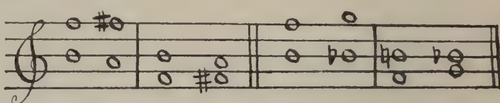
The Acentric operation involves the evanescence of contrasted equated elements by convergence on the Core, in which no contrast remains (beyond the "Hauptmann" conflict of the two Axes) owing to the Coherent blending to unity.

The Centricity is emphasised when the opposite cases are considered.

Taking the Hemichromes of the Matrix as the most neutral of its intervals, we note the ordinary Pythagorean resolution:—



and the opposite "resolution," which brings in the nonomial tones Fe and Ta, expresses Apocentricity with respect to the original Locus:—



The Envelope forms are defined by "Grades," the final "cadential" resolution is that of first grade (admitting the Contra-determinator).

The Core, as a Triad of definite pitch locus, and its Envelope, as a quadri-space about same in which any of the notes may be used, define the Matrical locus.

We have next to consider the processes whereby different complex groups are built up, and the contrasted process of "Catabolism" exhibited. This latter represents the convergence of such complexes into simple Tetrads of Radical or Symmetrical form.

The auto-axis of an arpeggio is obvious; figures of great variety being possible.

The axis and the clinear phonon are evident connections between successive tonal presentations, as extensions of identity in the duration and pitch continuities respectively.

Less obvious are the derived forms: Chordal Exchange and Sequence.

Chordal Exchange depends upon the audentity of tonal nominance within a definite chord (coherence), so that one tone can be exchanged for another without material loss of phonic identity.

In this way, the original " narrow " fluents may be exchanged for wider intervals of chromal dimensions without losing the character of continuity inherent in the original form.

The essence of chordal exchange is the establishment of a definite chordal progression and chordal coherence. Once these are firmly grounded the exchange can take place.

This property forms a test for chordal coherence, and cannot be demonstrated with any objective system of notation.

In notation, intervals of this kind cannot be distinguished from arpeggial extensions of chromes; hence at least one justification for a system of scientific terminology which will exhibit the difference has been made out.

The Sequence (in its simplest form, the Change Note) is based upon the presentation of a pattern progression (more or less exactly) on a continuous line of connection. This may be Scalar or Arpeggial, and secondarily a derived form.

The vitality of the device in modern musical works shows it to be no mere mechanical invention, into which the more elementary forms may easily degenerate.

SUBSECTION 3

PROGRESSION FROM A FLUENT ASPECT

A Fluent may be considered as an expression of relationship, corresponding objectively to the difference of interval between a pair of Chromes projected in a similar direction from the common tone which constitutes an Axis.

The discrimination of chrome and fluent is experiential: a chrome being a coherent entity in both simultaneous and successive presentations, while a fluent is an adherent entity in succession only.

Chromal relationships refer directly to an absolute experience, viz. the blending of series tones, but fluent relationships do not correspond to any simple experience.

A Fluent appears as an operative or nominative relationship between a plurality of whole or partial Series, and as such, necessitates successive comparison.

The fluental aspect of progression can be approached in various ways.

At present, it is pertinent to confine attention to the relationship of chord groups, and to defer other aspects involving "pitch-spacing," "inflection," "phonic continuity," "vocal identity," etc., till a later stage.

Fluents may be regarded, in a particular aspect, as differences of contiguous pairs of early series chromes, whose respective sums form chromes of a more elementary type than that of the components.

The practical advantage of this aspect is the facility with which fluents can be nominated from familiar chromes.

The respective sums and differences of contiguous series chromes are as follows:—

Components.	Sum.	Difference.
W + B	(W + B)	R
B + R	W	O
R + G	Y	IA
G + V	B	SA

This "summing" of contiguous pairs is an actual possibility, since by suitable means the experimenter can produce any desired chrome or compound of chromes; but the formation of a Difference is otherwise, and involves a preliminary transposition of the elementary chromes over a portion of the series in order to bring one tone of each to an axis of common pitch.

Fluence is thus seen to involve a manifoldity of acoustic elements, a plurality of originating series being implicated.

The Sum Chromes from the above table are seen to be concordant intervals, which the ear receives as entities in the shape of "chromes."

The Differences are certainly extreme chromes, but, with the exception of R (a point which will be discussed later), are jarring discords, whose experienced effect is that of a rapid alternation of the tonal components rather than fusion into an entity.

The predominating characteristic of a Chrome is unity in time. This characteristic inheres in both simultaneous and successive presentation of concordant intervals.

The predominant characteristic of a Fluent is axial unity; which is unity in succession, but incoherence in simultaneity, being thus distinct from chromality.

The basis of connection is multi-axial to an infinite extent.

Actually, a few predominant axes determine the type of each fluent, which in E.T. finally converge to the quasi-fluent R and the dodecanal fluents O and A, with their "inversions."

The most obvious condition of a step in pitch, from a chordal point of view, is the chromal relationship of both tones to some common axis.

This has been pointed out by musical theorists for a long time, and is found in the theories of Rameau, D'Alembert, Tartini, Zarlino, etc. But an idea of restriction to one axis has always limited the scope of the principle.

The fluents of greatest tonal determinance possess at least three primary tonal axes, which are followed by the infinite series of negligibles.

These primary axes refer to the relationships between the chromes V, G, R, B, M, Y, which occur in the diatonic scale, and which, in the dodecanal scale, fall into two groups of three contiguous concordant values, separated by the octave bisector, W/2, occurring between R and B.

When two tones are taken off one common tone, the difference projects beyond the common region.

Components.		Resultants.	Frequency Ratio.	Pythagorean.	Antinominant.
B + R	=	W	2		
B − R		O	9/8	1	7
2 B − W		O	—		
W − 2R		O	—		
R + G		Y	5/3		
R − G		I	16/15	1	5
2R − Y		I	—		
Y − 2G		I	—		
G + V		B	3/2		
G − V		S	25/24	4	4
2G − B		S	—		
B − 2V		S	—		
Y + B		W + G	5/2		
Y − B		U	10/9	3	9
2Y − (W + G)		U	—		
(W + G) − 2B		U	—		
Etc.					

This mutual space, in G. Boole's ingenious system, may be regarded as the "space" product of two directed co-linear quantities, and it is interesting to see in tonality an example of this ideal case.

The predominant discrimination between the axial sources may take the form of a limitation in consideration to the first-occurring concordant chromes.

The plus sign of the sum and the minus sign of the difference, as shown on previous page, correspond with the Coherent and Adherent characteristics of the early series-chrome relationships.

We have noted that the particular resultants illustrated above are the primary occurrences of that degree in the Series: the particular origin connotes a particular axis.

In the Nominantal aspect, an axis may be regarded as a bivalent component linking together the superimposed chromes, and in the operational aspect, as a pivot or axis upon which the progression of free tones turns.

Taking, therefore, the successive Series members as axes, their Sums (Perichromes) and Differences (Circumfluents) are seen to be as follows:—

Axis.	Adjacent Series Chromes.	Perichrome.	Circumfluent.
P	Z : W	W	W
P	W : B	W+B	R
P	B : R	W	O
P	R : G	Y	I
P	G : V	B	S
P	V : 7/6	7/5 (π)	—
P	7/6 : 8/7	R	—
P	8/7 : O	9/7	—
P	O : U	G	Comma (81 : 80)
etc.			

Regarding a Triad as primarily	P(3 : 4 : 5)
with adjacent chromes .	B/R R/G G/V
we have Perichromes . .	W : Y : B
and Circumfluents . . .	O : I : S

The primary axial relationships of a Triad are therefore expressed in terms of the three standard fluents O, I, and S, which with the ultra-oscillant U, form the steps involved in passing between any member of a Triad and a member of its envelope.

The difference between O and U and I and S vanishes respectively in E.T.

The "pseudo-fluents" involved by the recognition of a "just" contra-determinator do not now concern us.

The two Series being "directed" expressions, it is to be noted that the sum of Chromes is non-distributive, and thus determines the Species.

Thus $B+R=FW \qquad R+B=CW$.

The discrimination of summation vanishes in E.T. upon reaching the fluent degree of relationship $O+U=U+O$.

Thus, in Just intonation, the following ratios hold:—

$$L : T : LD :: 9/8 : 10/9$$
$$T : LD : TD :: 9/8 : 10/9$$
$$L : T :: LD : TD :: 9/8$$

This discrimination of polar predominance, either pole leading in the tertriad, vanishes in E.T.

This is another statement of the conditions of specified concordance, dividing the Series into its two principal sections, and it is seen that in just intonation the Supertonic of the Dominant differs from the Mediant of the Subdominant (necessitating an enharmonic change when modulating into these adjacent keys).

This seems simple and natural, and is accepted without question, but the result of continuing the principle leads to the Hexatonic Scale of Oscillants now used by some composers, in which each component is approached as of one order and quitted as another. Example:—

Doh	Ray	Me	Fe	Se	Le	Doh
Doh	Maa	Fa	Sa	La	Ta	Doh

D^0 (P : TT : D)
D^1 (P : TT : D)
D^2
D^{-1}(P : TT : D)

The Tertriadal fluents may be looked upon as the resultants of Inter-coherent (Triadal) and Phase relations; *i.e.* as compounds of the phase steps employed (Pythagorean and Antinominant) with variation in Order.

Upon the Pythagorean basis, it is seen that the phase translation is chromal, being of the first order, the only chromal aspect in the Hemicyclic Matrix.

The fluents between tertriadal members express relations between the horizontal and vertical rows of components of the Pythagorean Tertriad.

Determinator	LD : PD : TD
Tensor . .	LT : PT : TT
PRIME . .	L : P : T

whose comparative order is as follows:—

Second	Second	Second
First	First	First

(Zero) —one phase step-Centron —one phase step.

Since $LT = P$, and $TP = PT$, the fluent relationships reduce to those between the ratios:—

$$\begin{pmatrix} L : PD \\ L : T \end{pmatrix} \qquad \begin{pmatrix} P : TD \\ P : TT \end{pmatrix} \qquad \begin{pmatrix} T : L \\ T : LD \end{pmatrix}$$

together with the compound relationship of (L : TD), which may be looked upon as an extreme fluent.

These expressions correspond with the adjacent positions of the tertriadal components, when achromatically reduced to their close form, *i.e.* to the inter-relations of Centron and Polar terms, in alternate recurrence between the limits LD and TD, which presents the Matrix in the tertriadal type of Scale, as follows:—

Tensor Terms . .	TD	—	TT	—	—	—	—
Prime Terms . .	—	P	—	PD	—	T	—
Laxator Terms . .	—	LT	—	—	L	—	LD

The antinominant cycle presents Fluents as direct relations.

Oscillants may, therefore, be regarded as a neutralisation of species conversity, of similar character to J in the Pythagorean aspect, since, in E.T.:—

$$\pi T : \pi L = O$$

The progression between components of a Series (considered as a chord) is technically known as an Arpeggio, in the strict sense, as restricted solely to those components.

Change of matrical phase is seen to be translatory unless "equated" by an equal and opposite departure in chirality: in which case the Locus of the matrix is conserved.

This change of phase is regarded independently of temporal conditions, so that the appearance of definite polarity in con-

servation points to or implies the latent existence of its equating opposite, *i.e.* every polar component acts as part of the Envelope.

If either Core or Envelope be developed as an arpeggio, and the members of the Envelope or Core be respectively interjected in clinear order of pitch-occurrence, the result is the Septomial Scale, with its four types of fluents, viz.:—

Impellant 16/15, between (TD : P) and (PD : L).
Infra-infra-infra Oscillant O''' (9/8), between (P : TT), (L : T), (T : LD) from T aspect.
Infra-infra-ultra Oscillant $O''^{\wedge}$ (10/9), between (TT : PD) and (LD : TD) from L aspect.
The Neutral Oscillant G/2, occurring between the Scalar Limits (TD : LD).

The consideration of the Septomial Scale as between the limits TD and LD may appear somewhat strange, from a conventional standpoint. It is, however, only a theoretical abstraction for the purpose of convenience.

The general form of the scale, in Tertriadal type, is as follows, the centron determinator being regarded as tervalent:—

FUNDAMENTAL ACLINEAR

			Me			
Te						Lah
	Doh	Ray	Mae	Fah	Soh	
Ta						La
			Ma			

COINCIDENTAL DECLINEAR

			Doh			
Fah						Soh
	Me	Ray	Doe	Te	Lah	
Fe						Se
			De			

The Hemicyclic Scale, on the same arrangement, appears:—

Pythagorean terms.—Quintensor, Prime, Bitensor, Quadritensor, Laxator, Tensor, Tritensor; and can be put in any terms, so that only Pythagorean expressions appear.

In "Neutral" form it appears in terms of the Yoke, as follows:—

$${}^{F}_{C}\left(J^{4}, J^{-1}, J^{1}, J^{3}, J^{-2}, J^{0}, J^{2}\right)$$

The Seriopolar Scale may be set out also in the respective origin terms, reduced to the nearest arrangement to correspond.

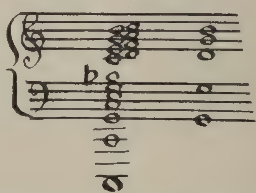
The Septomial Scale divides into two Semi-scales, corresponding to the Greek Tetrads, which may be put into the two arrangements: (1) having a common centre tone, (2) adjoining, but separated by a " disjunct " interval.

The " disjunct fluent " between the terminators TD and LD (Te : Lah) and (Fah : Soh) is neutral in Species, but is determined solely by its relative locus in the " general " scale. It is permutably variable from A to V in dimensions.

The nearest approach in experience to the natural generation of fluents occurs in the harmonic series under conditions somewhat as follows:—

Suppose a " series column " on a given prime to be so modified that the prime and its octave vanish, the effect will then be a peculiar " klang " in which the third-series member is predominant.

If now this experience be compared with the natural harmonic series on this particular member (a comparison which would naturally occur), the transition would be as indicated.



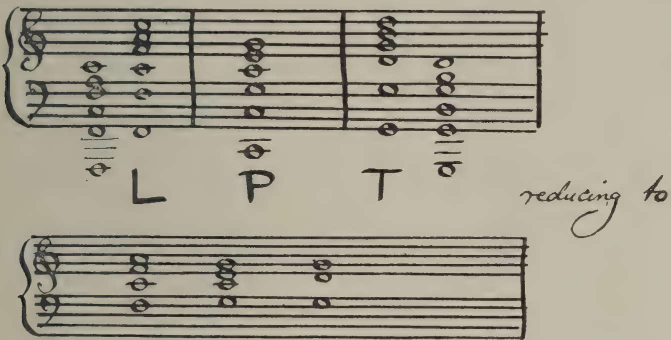
Supposing that conditions favoured the comparison in succession, the change of tone-tint as a unity would be nonchordal.

However, to an ear perceiving the harmonic elements, and the actual case would tend to emphasise them, the effect would be distinctly chordal, and the adjacent upper tonal members would be appreciated as forming possible " fluents."

The most natural case of this kind would occur in a peal of bells, where the " natural fluents " are very evident, *once the ear is directed to them.*

Reverting to the example, suppose the Series transposed achromatically. This change, which is not of unusual occurrence, gives rise to the same colligation of audental components, as

well as to certain natural axes, the early series members being sufficiently prominent.



In this somewhat artificial case the original "clarifying" of a mass of upper partials into simpler triadal forms becomes transformed to a definite "polar" triadal progression, in which the early fluents are apparent.

Upon continuing the process the lesser fluents also appear, but the natural conditions of audentity and perception do not favour their recognition.

In this aspect of the case, fluents appear as conversions of adjacent tones in the two series columns, consequent upon Pythagorean translation of a Series.

The actual chord forms are of an euphonious character, and the "oscillant" tones occur in the gaps between the axes, and so do not conflict with them.

It will be noted, however, that the perception of separate series components in the successive series, which is necessary before any effect of "natural fluence" is evident, is inseparably involved in the apparition of Consecutive octaves, fifths, fourths, etc.

These, if so perceived, are dissonant progressions, but when occurring unindividualised in the general change of tone-tint, do not affect its euphony.

In blowing the "natural" series of tones on a horn, etc., the tones themselves are compound, *i.e.* series, and if their separate components were noticed as a column of independent tones, there would certainly be an effect of natural polyphony.

But at the same time, the natural dissonances due to parallel first-order chromes would be noticed, and the effect would be cacophonic.

The analysis and separate perception of series tones is an artificial process.

From the fact that historically nothing of the kind was described until after the experiments of Mersenne and his followers had paved the way, it may be safely concluded that the experience was unknown to humanity before.

Hence " natural fluence " cannot have been evident, and consequently the construction of scale forms owes nothing to it.

Even at the present time, the experience comes as something new to the majority of people, when attention is first drawn to it, and the Seebeck *versus* Ohm controversy of the forties shows how experienced investigators may vary in their views on the phenomena.

The four tones of the enveloping Tetrad plus the three tones of the Core, put into Tonic-Solfa terms, appear as the following groups:—

Tones.	Fundamental.	Coincidental.
Doh	Core Prime	Core D
Ray	Tensor-tensor	Tensor T
Me	Core Determinator	Core P
Fah	Laxator-prime	Tensor D
Soh	Core Tensor	Laxator D
Lah	Laxator determinator	Core T
Te	Tensor determinator	Laxator P

The horizontal lines mark the loci of the disjunctive fluent, whose progression does not involve change of order unless commutation occurs.

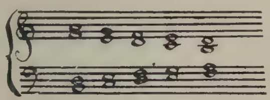
The " Trinomial " of Conserved Envelopes, in Hemicyclic type, appear as follows:—

L	P	T
Ray	Lah	Me
Te	Fah	Doh
Soh	Ray	Lah
Me	Te	Fah

From the above, the components of each Triad can be extracted as compound chords, although identical as to tones with the three coherent triads of the tertriadal matrix.

The Tetrad Envelope is seen to consist of a pair of Polar Dyads, whose "resolution" on to the contained Core triad involves motion in opposite directions.

It is interesting to note that the continuation of this motion achromatically reproduces the envelope by transposition of the polar elements.



L to T

T to L

This is the basis of Tetraphonic progression, or Scale duality, which is involved in the conservation of polar equation about the Centron.

Further discussion on this point follows in "Successive Tonality."

The relation between the extreme tones of the Septomial scale in either species is evidently "dis-junct" or non-axial.

This apparent gap is axially bridged by the fact that the Yoke in dual concomitant species is Tensor to the Tensor triads of both species.

The progression is axial, and each chord is included in the Envelope (Pentadial form), the moving notes form "phona" bridging the disjunct.

It should be remembered that the Yoke is contra-determinator of CP and FL◖. Should, however, the chromes in question occur in the Laxatorial Triads of both species (concomitant), the progressions are not only non-axial, but the inherence in a pure envelope is not evident.

The Concomitant Laxator triad appears in the nominant species as a compound of polar and core element, viz. as $(PD : \frac{PT}{TP} : TD)$.

It is therefore neither a pure core nor pure envelope presentation, but a transitional seminomial.

On the other hand, it is in itself a coherent triad.

The contrasted elements stand in the relationship of a first-order chrome to each other.

As long as these two aspects are kept separate, no particular effect arises; but, by putting the two "concomitant" laxator triads in comparison (by succession in presentation, or otherwise), these two opposed aspects are brought into collision, each triad appearing in the concomitant aspect as a compound chord.

This effect is well known in experience as the "False Relation of the Tritone."

Its bearing in the present case is to differentiate between the conjunctive Tensor relationship, and the disjunctive Laxatorial colligation, in the Concomitant Species.

The distinction has really a more basic foundation in the conditions under which the discord is accepted in Tonality, viz. as a time presentation. This aspect may be briefly considered.

A seminomial triad, such as the "recessives" of any Matrix naturally appear, can be regarded in two ways.

In the older modal system, the only chromal connection was that occurring in the Pythagorean relationships, viz. First order. The distinction between "species" thus was not recognised, and all six triads were treated upon equal terms.

But with the synergetic evolution of the modern tonal system, possessing Second-order chromes of distinctive species, quite another set of experiences were developed.

In the purely modal system there are no "envelopes," and the limit of triadal connection is attained at the "imperfect triad." Even this hiatus was bridged by the device of converting *b*-quadratum into *b*-rotundum, which made progress in a laeval direction possible (the opposite method of making the tone Fah bivalent does not appear to have been evolved till much later).

Consequently the distinction of Recessive and Predominant does not appear, and can have no significance in modality.

It may be mentioned that the methods of Modality, with its restriction to first-order coherence, are quite as available for use to-day as they were in ages past, and many highly artistic effects can be produced thereby.

We can go back to the older "republican" methods but we cannot re-transform our ears to the same state; consequently, it is difficult to avoid judging modal progressions from tertriadal

standards, and hence, to the lay ear, modal progressions have a strange archaic touch ill according with the old title of "Plain"-song.

(Possibly, in the future, ears attuned to contra-second-order chromes may look back upon our harsh thirds and sixths, and wonder how we put up with them.)

To modern ears, the Recessive triads sound as discords because they are felt as compounds of triads. The Concomitant Tensor triad sounds as part of the Nominant Envelope, and the other two triads as Tensor and Laxator "Seminomials."

Since the time of Monteverde, the modern ear has evolved the perceptive acceptance of discords as "things in themselves," *i.e.* as Envelopes, or compounds of Envelope and Core.

The difference between the Modal and the Tertriadal aspects is in the acceptance of the Fluent, which is a tonal element involving progression of both phase and order. The only "fluents" possible in the Modal system were those between Zero and First-order elements.

Ancient nations could have produced a "Tertriadal" scale by interpolating the tones of three instruments of the horn class.

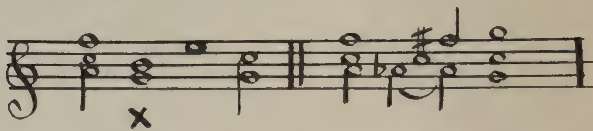
The fact that they did nothing of the kind is evidence of the lack of subjective basis.

The Fluent, considered as a fluent apart from a small chrome or large limen, is a definite product of Synergy.

It has already been mentioned how the Chrome itself became a synergised entity by the actual use of the compound effect already due to the associated tones; the process being stimulated by, if not actually due to, the availability of instruments of the keyboard type (whereat a musician could perform direct solo chordal experiments instead of relying upon the indirect processes of assembling a choir, orchestra, etc., formerly necessary).

Similarly, the associated compounds of notational fluent appearance were available through the recognised method of preparation and resolution of discords, as Time Stages in the progression from concord to concord.

Monteverde only confirmed the tendency of his age by boldly taking these compounds, already "synergised" by use, and treating them as entities, and he was vigorously attacked for so doing, but was no more culpable in principle than his predecessors who had similarly utilised the chrome.


etc.

As the Chrome, although used as an entity, still carries the implication of its located serial origin, so the Fluent, even when used as a simultaneity, yet retains the Time-succession character due to the source from which it was evolved.

This is the characteristic difference between the Fluent as such, and the same notational configuration of tones regarded as extreme Chromes or Limina.

The question thus naturally arises as to whether the Limen will in the future be subject to the same process of evolution, and give birth to an individual synergised entity, thus placing a new materia of expression at the disposal of the tone artist.

It may be recalled that the Limen is approaching the limiting dimensions of auditory discrimination, and the further physiological evolution of the ear, although probable in the dim future, cannot rationally be of comparative interest in the present connection, which only deals with processes concerned in the last thousand years or so.

Supposing, for instance, that such instruments as the quasi-mechanical players of to-day are constructed on some of the proposed systems to give just intonation in any key played in.

This is only a question of expense and presents no difficulty beyond those incidental to careful tuning and maintenance.

In that case, the converting limina between one "just" key and another will have real significance apart from the mere mistunings or tempering into which they now merge; and therefore may become "synergised entities" (enharmonies?) for the future musician to exploit.

This is, of course, mere speculation, but not without some interest.

Some classification of progressional relationships is afforded by the core-envelope view.

Progression from a Core to an Envelope, wholly or in part, may be known as Projective, and the converse Retractive.

Other progressions are of a neutral type.

A Core triad, or any of its members, may be considered as a Zero Envelope and symbolised by the expression E^0.

Its Envelope, or any member, is the Unit, symbol E^1 or E.

Secondary, Tertiary, etc., Envelopes follow:—E^2, E^3, etc.

A Secondary, Tertiary, etc., Envelope may be either Conserved (Tonal) to the Septomial, or Translatory (Real).

The translatory type involve extension of the Matrix as follows:—

Primary	Envelope	Septomial	Tertriad
Secondary	,,	Nonomial	Quintriad
Tertiary	,,	Ondecanomial	Septriad
Quaternary	,,	Tridecanomial	Nontriad

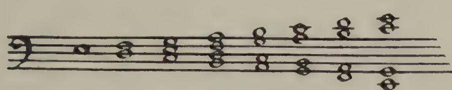
The last type merges into the E.T. Dodecanal Range.

The Conserved type may be shown in tabular form:—

Core. E^0	Primary. E^1	Secondary. E^2	Tertiary. E^3	Quatenary. E^4	Quinary, etc. E^5
					PD
			P	TT	P
		TD		TD	
	LD		LD		LD
T		T		T	
	L		L		L
D		D		D	
	TT		TT		TT
P		P		P	
	TD		TD		TD
		LD		LD	
			T		T
				L	
					PD

All the determinators being considered permutable.

The rationale of this classification is applicable to the " chord " forms occurring in the Tetra-phonic progression, as Enantio-diphony:—



E^{-2} E^{-1} E^0 E^1 E^2 E^3 E^4 E^5

where the successive forms are obtained by addition or subtraction of units to the index, the Core Triad being taken as Zero.

Theoretically, Envelopes with negative indexes may be used. By an extension of the Synergetic process, all the " forms " thus occurring may be accepted as chords of varying audentity.

SUBSECTION 4

PROGRESSION FROM AN ANTINOMINANTAL ASPECT

The antinominantal aspect of Fluent progression has a peculiar acoustical interest of its own, due to the fact that the experience of Parasyntony (Liminal adjacence) is the rational element in its formulation.

The Antinominant appears as a step in the dodecanal cycle of this type, the period being the Octave.

It also appears as a Tetragonal (orthogonal) step in the cycle of the infra-second-order chrome G, which approximates to the Parasyntonic externominant.

In the latter aspect, an Oscillant bears the same relationship to an Antinominant as the quarter-dodecanal does to the Hemicycle, representing a total neutrality of Species.

To understand the bearing of this point, let us consider the three tones Te : Doh : Ra as loci of key.

Using current musical terminology, in this case we pass from a key of five sharps to one of five flats by translation of locus from Te to Ra by way of Doh, thus the oscillant step is of neutral polarity. Similarly, if the key of six flats or sharps be taken as the " diametrical of the natural basis " the same result is obtained, but the operation is no longer Antinominantal.

The tone nomials of the Antinominant cycle may be extended to Triads, whose respective " orders " bear the same relationship to each other. Thus, with the important distinction as to non-identity of Grade interval with the first-order component chrome of the triad, expressions similar to Pythagorean Tertriads are obtained.

It is to be noted that the alternate coincidence and diametrical relationship between the members of the Pythagorean and Antinominant Cycles, limits the differentiated expansion of Nonomial-triads to odd groups.

It is also to be noted that the ultra-second-order steps, and the diametrical of the origin, form the meeting-place of both types.

The effect of the "odd" coincidences merely inverts, with respect to the octave, the derived intervals, viz. G to M, O to (W – O), while V, Y, and W are common to both cycles.

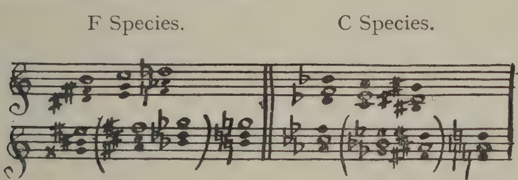
The antinominant range of variability is restricted to six E.T. steps in either direction, which, with respect to origin of species, are arbitrarily projected upwards or downwards from the basis upon the range of absolute pitch.

The laeval and dextral projections meet in the semi-achrome πN, diametrical in respect to the basis tone.

In considering the Tertriad of antinominantal type, there are two forms presented, on the odd and even basis respectively:—

$$\pi L : \ P : \pi T$$
$$L : \pi P : \ T$$

In Staff notation these forms are:—



These illustrations have been written out in the form they are most likely to be met with as progressions, viz. as "first inversions."

The open note signifies the Nomial prime.

The chords, in Pythagorean terms, are seen to be compounds of quinomial type, comprising tones of the extreme "secondary envelopes" in nonomial terms, thus maintaining the equation of polarity about a centron.

They represent the extreme extension of conservation, and thus the point at which the Antinominant and Pythagorean matrices meet and mutually terminate each other.

Progressions of the "Augmented-Sixth" type are distinguished by their chromatic components, and their progressional implication is that of the contrary motion of antinominant fluents.

Their derivation from simultaneous configurations of chromatic appogiaturas or passing notes is obvious from the simplest form under which they appear. This has been pointed out by Sir Hubert Parry and other investigators.

Some of these cases may be illustrated here.

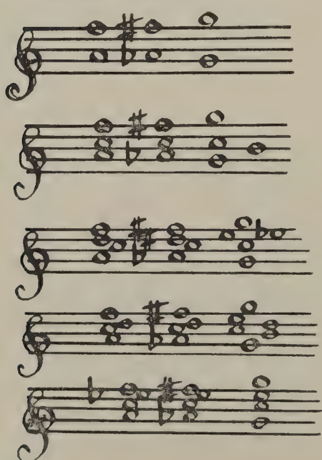
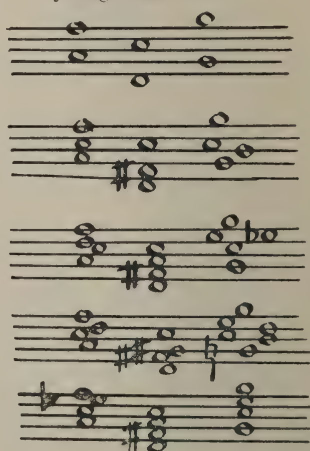
F Species:—

C Species:—

The relationship of the triads of πP to those of PL and PT is seen in the following example:—

To Laxator. To Tensor.

It is interesting to compare these antinominantal progressions with the corresponding Pythagorean forms obtained by the diametrical transposition of the Polar chords.

Antinominant Form. Pythagorean Form.



The progression known as the "Neapolitan Sixth" is considered by Helmholtz to be a chordal statement of the old Mode

known as Greek Doric, or Ecclesiastical Phrygian, and this is one of the facts put forward in support of his theory that our present Minor Mode is a survival of fragments of the older modes.

Antinominant Form. Pythagorean Equivalent.

Transposition of Matrical Locus, either for a short or considerable extent, over an Antinominant Fluent in either direction, occurs so frequently in practice as to render illustrations unnecessary.

The diametrical transformation, in Fundamental Species, may also be noted, between:—

Antinominantal Form to Pythagorean Form.

The point to be noticed is the correspondence between the extreme chords of the Pythagorean Form with the simplest chord relations in the Antinominantal.

The converse is not so notationally evident, but applies with the same force.

This mutual limitation of each other's chords, limits the theoretical infinity of chordal extension, with its corresponding notational complexity of double sharps and flats, to the E.T. Dodecanal terms.

At the half-way point between each origin there is a region of relationships which may be said to be the neutral meeting ground of both aspects.

These are the tetragonal and trigonal chord relationships.

In practice, Antinominant chord construction and progression is much less frequent than Pythagorean.

This is largely attributable to the fact that Pythagorean gradation is itself chromal, and thus identical with one of the elements of Triadal construction—the first order, which is of greatest

audentity—whereas, in the Antinominant case, the converse prevails, sharply distinguishing the " nomialism " from the triads.

In addition, neither the semi-graphical Staff notation nor the symbolic Solfa system presents the antinominant relationships in a simple light; and since instrumental control is in accordance with the Pythagorean system, the execution of antinominant progressions offers some obstacle to facility. These same factors, as we have seen, also militate against the hexatonic scale of oscillants.

The contrasted directivity of the determinator and contra-determinator in the same partial or total species is evident from chiral considerations.

This is not always obvious in a system of notation which makes no distinction between notes representing the two classes of tones, but the student of harmony is led to recognise the essential though subtle difference between, for instance, a " third " and a " thirteenth."

For the rest, we shall have to wait until an E.T. system of twenty-four notes to the octave (bi-dodecanal) permits of the contra-determinators being more justly approximated by the " quarter-tones," but this, of course, introduces the old question of distinction between enharmonic transformation and tempering accommodation.

The oppositely directed character of D : Ɑ is seen more distinctly in " Successive Tonality," where the association between determinance and chirality is examined.

In the simultaneous aspect, the principal phenomenon is the contrast between the hemichrome W/2 formed by the inversely directed terms D : Ɑ, and the First-order coherent chromes B and R to which it approximates.

CHAPTER XII

SUBSECTION 1

CONSERVATION

The nature of a Matrix, and the conditions of its conservation, form an important part of that unwritten knowledge possessed by every musician.

The expression of the character, conditions of existence and persistence, of a conserved matrix, can be set out in determinate tonality by various methods that facilitate not only the definite knowledge, but also pave the way to a comprehension of progression generally, and Translation of Matrical Locus in particular.

There are many ways of approaching the subject, but it is convenient to restrict present examination to the Phase aspect, to set out the Matrix and its range of variability in cyclic diagrams and thence to obtain limiting conditions in a geometric language which can be expressed in terms of tonal relationship.

This method, although apparently open to the accusation of artificiality, is chosen because the detailed examination of tonal relationship by other methods is necessarily dealt with when considering translation.

The concise method of graphics can most conveniently precede detailed discussion, and is reliable to the extent that should any error or sophistication enter, its detection cannot evade the logical conclusions which follow.

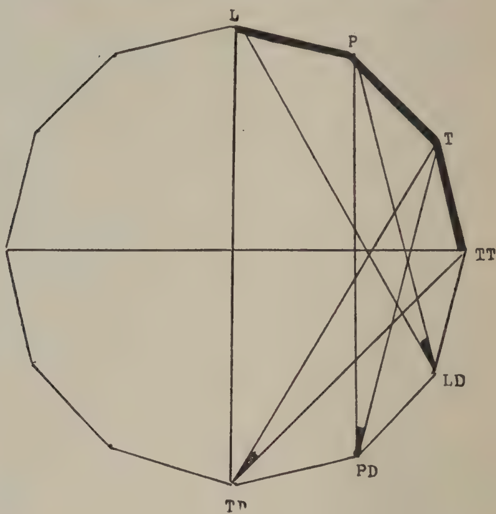
The graphic method has the advantage of presenting relationships *en masse* to the eye, and thus facilitating the realisation of what might otherwise appear somewhat disconnected abstract statements.

The aspect of the Septomial Matrix as a limited equation of Polar terms about a Core has already been discussed.

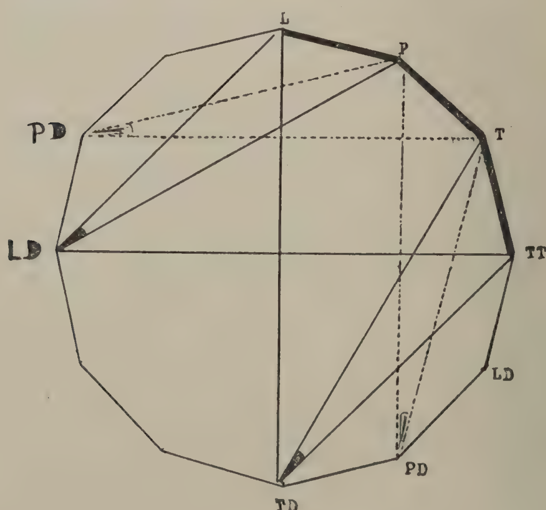
The three typical experiential aspects may be put into form, and examination be directed to the co-ordinate constitution of chiral direction.

The analysis of the Septomial Matrix into two adjunct semi-

Hemicyclic Tertriad (F Species)



Symmetric-Composite Tertriad (F Species)



matrices (apart from the Core-Envelope aspect) may also be effected, and shown in similar terms.

The triads are shown by triangles, whose direction points are the determinators, the species being obvious therefrom.

The diagrams are drawn to illustrate the Fundamental Species. By viewing same in a mirror held beyond the diagram parallel to the (J : πJ) diameter, the concomitant coincidental arrangements can easily be seen.

In the first of these diagrams, it is seen how the Matrix is divisible into two quadrantal semi-matrices; the nominating quadrant being the predominant component, contains the three zero-first-order " bases " of the triad, the tones being " univalent " and the chromes forming units of cyclic grade.

The " recessive " or " concomitant " semi-matrix contains the bivalent elements, viz. the permutable determinators.

The second diagram shows the symmetry of the tertriadal form.

Since the theoretical symmetric prime determinator PĐ is not shown on the cycle, the alternative composite types are given in dotted lines.

The two diagrams represent the Seriopolar type of Matrix in Tensor and Laxator series respectively.

The group of seven members has been divided into three Quasi-triads (the first being exact) as follows:—

L (1 : 5 : 3) (3 : 7 : 9) (9 : 11 : 13)

and the diminution of average audentity is indicated by drawing the triangles with three, two, and one lines respectively.

The apical direction shows species, as before.

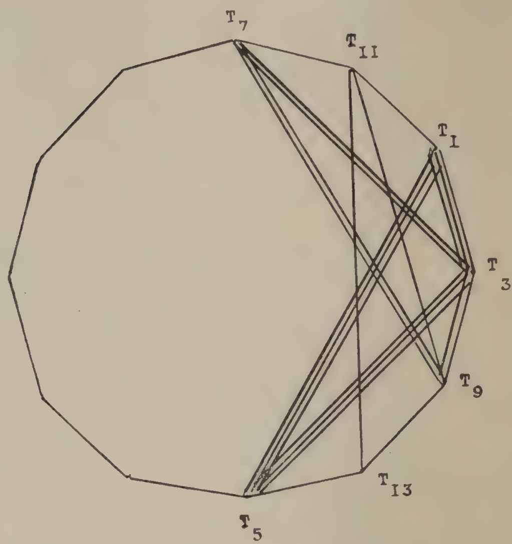
The seriopolar type has little effect as a whole on the conception of a septomial matrix, in comparison with the Tertriadal and Hemicyclic bases of arrangement; but the coherence between the members, especially in the sub-groups and radical elements, is a real acoustical factor, and, though the approximation of the " further " members, 7, 11, and 13, to the terms of the other types is somewhat loose, it is seen that the Polar Series (especially that of the Tensor) is sufficiently near to warrant the abstract consideration in the Fundamental species.

It is certainly closer than any other series.

The " tangent " criterion of regionality in Phase has already been considered in connection with Order.

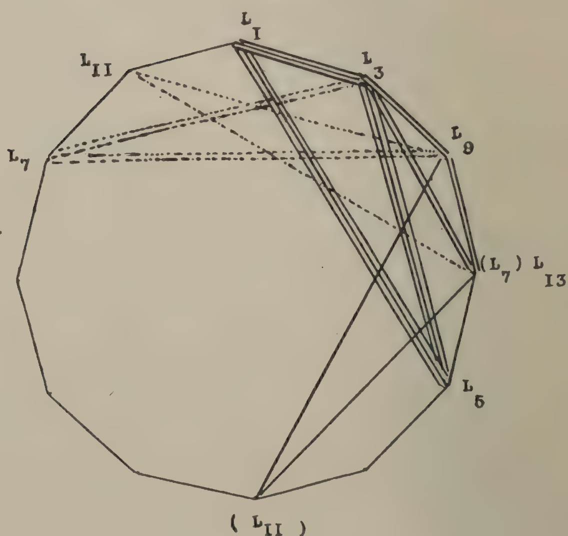
Tensor Series

$\Sigma_1^{13} T$



Laxator Series

$\Sigma_1^{13} L$



It may be regarded more as an illustrative device rather than a definitive expression of relations, but it is certainly convenient, and facilitates comprehension through the eye.

The chiral direction in which the tangent of a hemicycle "heads," extends between N and πN, *i.e.* between the limits of complete sign reversal at the horns TD and L.

The criterion of the hemicycle is thus chiral irreversibility whose transcendence introduces diametrical replicas of existing terms into the matrix, which is plurality.

The symmetric matrix is a bi-specified (or rather ambi-specified) expression.

Hence, the Hemicyclic and Composite types of Tertriadal Matrix may be looked upon as extremes of one species, maximum and neutral, which are thus capable of delineating a range of matrical types all satisfying the conditions of Vertication.

Colloquially, it could be said that the "minor mode" is the most "fundamental" way of presenting and utilising the constructs of the abstract Coincidental Matrix, without loss of the latter's concordance and specific character.

The semi-matrical quadrant, or quadrinomial, may be similarly viewed by the aspect of its chiral tangent, which sweeps between zero and unity $\pm\left(\frac{\pi}{2}\right)$

The implication as to the "orthogonal independence" of the Orders has already been discussed; the evanescence of an orthogon marking the transition of Order.

The constitution of a matrix as composed of Triads, and discriminated into two portions, one (predominant) containing the bases or nomial units and the other (recessive) the species-directed apices or determinators, is a useful conception to have in mind, since it enables discrimination between commutative and non-commutative translation to be considered in terms of independent order.

Antinominantal Translation is effected by the diametrical translation of alternate Pythagorean terms.

There is no corresponding case in commutation, since the diametricals of tones in Second-order chrome relation are of the O and V type, thus being at the limit of the parasyntonic tract, while those of First-order related tones are antinominants, and appreciably within each other's theoretical tracts.

This is the only basis upon which antinominant relationship is rational.

SUBSECTION 2

TRANSLATION

Translation may be defined as the movement of a Matrical Locus in pitch, in distinction to its static position in conservation, within a more or less restricted domain.

It will be seen that this definition practically corresponds with the technical meaning of "Key modulation." It is, however, inadvisable to employ a term which, though perfectly suitable for empirical description, is affected by conditions such as those of notation and performance, etc., extraneous to the abstract aspect.

Retention of technical names being liable to lead to misconception and narrow applicability, the term "Translation" may be selected to express the abstract aspect of matrical movement.

The description of forms and types of Translation is inherent in that of Matrices themselves.

Broadly, Translation involves the substitution of a transitional bivalence in the elements as they pass from one monovalent matrix to another.

This state of bivalence or ambiguity is, however, restricted to the connection between the two definite matrices, and is differentiated from conservation.

The translation is marked by the transitory bivalence of the components, *i.e.* inherence in the original and succeeding matrices. This state of bivalence is restricted to the two matrices and may be very prolonged and circuitous. Throughout the process, any case between the extremes of premised and ambiguous destination may occur, but the determination is never obscured. Translation effected by an intervening chaos of indeterminance is not unknown in practice, but cannot be regarded as music: partaking more of the nature of Tonal hypnotism.

Consideration necessitates examination of the following aspects:—

(1) The possible ranges of Translatability.
(2) The conditions of plurivalence inherent in the axes and fluents of progression.

(3) The directive agencies.
(4) The limiting factors.
(5) The equation of elements within the dodecanal matrical range.
(6) The ineliminable condition of Vertication.

Since any set of tonal relationships (comprising matrices) are expressible in terms of Degree, Phase, and Order, any movements must be capable of statement in similar terms.

Phase is a definitive factor in any question of relative locus, and forms the criterion between a locus expressible by the relative position of a matrical centroid, and its range of variation possible under determinal circumstances.

Order, as an expression of co-existible independence, must necessarily become plurivalent in total or partial bi-specification of terms.

Degree, however, being based upon acoustic criteria, retains its univalence throughout translation, except in the comparatively restricted regions where definity merges; such as the superfluent-hypochromal values between V and O.

The expression of conservative stability naturally precedes any discussion upon translation, thus involving some apparent recapitulation of matters already discussed.

If conservation were regarded as a rigid characteristic of matricality, recognition of translation would be precluded. Examination must, therefore, be directed to the potentially unstable elements and factors of each type of matrix, under varying conditions of environment.

If, on the other hand, the equations of tonality, upon which conservance rests, were regarded as mere casual bonds, which could dissolve in all directions, conservance as a determination of locus would cease to exist.

Definite conservance, and an apparently unrestricted fluidity of translatability (except possibly in the diametrical case), are recognised as coexisting in all cases of experience.

Unless this fact be granted, it is difficult to see how determinate tonality exists, and impossible to admit its evolution from the known facts of history.

A preliminary classification of translational types presents three broad groups:—

I. Total and Partial Commutation of Species.
II. Revolution in Phase.
III. Parasyntonic substitution.

These are seen to merge into one another, since partial commutation involves in its three orders achromatic rearrangement, reversion of polarity, and permutation (the last named is a particular case of parasyntonic substitution).

Of the commutational type, two cases are predominant, viz. those in which the binaxial dyad is respectively of First and Second order, the fluent elements being complementary thereto.

A nominant matrix of given species has, therefore, two commute comparisons attending it, which are respectively the Permuted and Concomitant types, both of same species, but differing in locus by a phase quadrant whose hemicyclic chirality is dextral for a fundamental and laeval for a coincidental nominant.

This introduces the second class of translational type, viz. Revolution of Locus around the Phase Cycle in a definite chiral direction.

For convenience, such relationships may be expressed in Pythagorean terms, but this, of course, does not preclude any other system being adopted.

The translation is then measured in definite Dextral or Laeval steps from an arbitrary origin, the steps being expressed as fractions of a whole period.

This type divides naturally into five classes:—

I. Adjacent or uni-grade steps of First order.
II. Trigonal and Tetragonal steps of Second order.
III. Bi-grade or Oscillant steps, intermediate to the first and second classes, which are non-axial.
IV. Quinti-grade, or Antinominant steps, which are treated as of the third, or parasyntonic, type.
V. Diametrical or Sexi-grade steps. These latter form the limiting condition, since the distinction of chirality vanishes in the E.T. equation $T_{VI}=L_{VI}$.

While not promulgating the indeterminance of this latter class (for cases are found in musical works, though rarely), it will be seen that the axis involved (πW) is a component of the neutral-species Envelope, and does not exist in the Triad.

Hence this class of translation must be of essentially different character to any other.

The third of the Translational types, viz. Parasyntonic substitution, differs very considerably in character from the others.

Whereas both commutation and Pythagorean types refer directly to axial coherence in the Series, antinominant relationship rests upon another acoustical basis of connection, viz. the region of co-stimulus surrounding a locus of maximum resonance in the basilar system of the ear.

Hitherto, this type has not received much attention at the hands of empirical theorists, who are often fettered with practical considerations based upon the predominance of the other two types, but the experiences themselves are none the less evident in musical works.

Somewhat different language may be necessary to describe the conditions peculiar to a system of connection which does not depend upon a tonal axis, and in which, consequently, the fluent is paramount.

The nearest relationships on the antinominant cycle are the diametric Pythagorean polars.

The axis, when it appears, is a permutable contra-determinator which is added to the triad, and thus the axial aspect is necessarily tetradial and consequently of a different standpoint to that usual in the Pythagorean aspect.

This will be seen from the progressions between Core and Polar Radical Tetrads.

Infra Core Tetrad to Ultra Laxator Tetrad.

P◖	axially changes to	πLℂ
T	moves one antinominant step to	πP
D	,, ,, ,,	πLD
P	,, ,, ,,	πL

Ultra Core Tetrad to Infra Tensor Tetrad.

Pℂ	axially changes to	πT◖
T	moves to	πTT
D	,,	πTD
P	,,	πT

which may be put into Solfa symbols:—

Fundamental Infra Core Tetrad to Ultra Laxator Tetrad.

$$\left.\begin{array}{l}\text{Ta}\\ \text{Soh}\\ \text{Me}\\ \text{Doh}\end{array}\right\} \text{ to } \left\{\begin{array}{l}\text{Ta}\\ \text{La}\\ \text{Fah}\\ \text{Ra}\end{array}\right.$$

Coincidental Equivalent.

$$\left.\begin{array}{l}\text{Fe}\\ \text{Lah}\\ \text{Doh}\\ \text{Me}\end{array}\right\} \text{ to } \left\{\begin{array}{l}\text{Fe}\\ \text{Se}\\ \text{Te}\\ \text{Re}\end{array}\right.$$

Fundamental Ultra Core Tetrad to Infra Tensor Tetrad.

$$\left.\begin{array}{l}\text{Lah}\\ \text{Soh}\\ \text{Me}\\ \text{Doh}\end{array}\right\} \text{ to } \left\{\begin{array}{l}\text{Lah}\\ \text{Fe}\\ \text{Re}\\ \text{Te}\end{array}\right.$$

Coincidental Equivalent.

$$\left.\begin{array}{l}\text{Soh}\\ \text{Lah}\\ \text{Doh}\\ \text{Me}\end{array}\right\} \text{ to } \left\{\begin{array}{l}\text{Soh}\\ \text{Ta}\\ \text{Ra}\\ \text{Fah}\end{array}\right.$$

It is necessary to be very clear as to the concept of Conservation, apart from the particular aspect of a matrix as an equation of system about a centroid; otherwise it is a loss of labour to proceed with the investigation of translational conditions.

It may be laid down as an empirical axiom that there is no indeterminance in conservation, although the boundaries of system may vary in independent cases.

The recognition of a locus is practically the most persistent element in tonal experience, even if the system of notation be not capable of clearly showing it.

Univalence of Species follows as a necessary extension of this axiom. This is exhibited by distinction of degree, chirality of phase, and definity of order, which stand as partial abstracts of Species.

Determinance of Degree is evident in the restriction of Triadal and Tertriadal components of the Series.

The very condition of the E.T. system, which permits closing of the phase cycles, involves a definite distinction between the Radical Triad Σ_1^6 and Radical Tetrad Σ_1^7, the former as a potential Core-locus, and the latter as a polar moiety of an Envelope; the two forms being contrasted components of the located Matrix.

The univalence of Species is manifested in the Core Triad. A symmetrical triad is an acoustic possibility but a tonal impossibility, since it is a discord, and therefore not essentially discriminated from an envelope.

Plurivalence of Species is an essential characteristic of the

symmetric envelope about a triad of either species, and the wide range of translational possibility is largely indebted to the indifference of core species, the same envelope being utilised in both.

This condition appears in partial species.

Certainly it would appear that Zero partiality (achromatic arrangement) was independent of the condition of univalence, but even in this case such elementary experiences as the particular characteristics of " second inversions " reveal a differentiation.

Other phenomena of the same type are those of the uninvertibility of the interval F(Tᗡ : Lᗡ) and the permutation of the polar determinators in the symmetric envelope towards the direction of progression, giving rise to the two varieties of " Minor " (melodic) Scale, Ac- and Declinear.

This latter phenomenon is of sufficient importance to be deferred for fuller consideration.

As regards First-order Partiality, the essence of mono-chirality is evidenced by the rare use of diametrical translation in which chiral distinction vanishes.

It may be pointed out that this is one of the factors which the use of Just Intonation would obviate, in theory, at least. It is possible in E.T. to devise configurations differentiating T_{VI} and L_{VI}.

As regards the Second-order Partial Species, a triad is either definitely Fundamental or Coincidental.

The so-called Gypsy or Picardian Mode (Hauptmann's Major-Minor) is seen to be merely one of the composite methods of presenting the ideal symmetric Core in E.T. terms.

In this connection the rationale of Septomial limitation is seen.

The next extensional form of the Matrix is the Nonomial or Ter-heptad.

This contains the tones Fe and Ta (which are in F species respectively and in C species contra-respectively), TTD and LL, the latter being equivalent to T◗.

These are permuted determinators in nominant and concomitant triads of the Tensor.

Since no system can possess triads having more than one simultaneous determinator, the plurivalence of determinatorial species, in any matrical form beyond the Septomial, definitely limits that system as one of chordal simultaneity.

Consequently, the experiential boundary between Matrical

Form and Matrical Range is very definitely determined at the Septomial by the necessary univalence of the second-order tonal elements.

Since a minimum of two definite matrical loci is involved in translation (as precursor and successor) independently of epoch of occurrence, or simultaneity as regards component progression, it is seen that a linear axis is substituted for the Centroid of equation, and further, that a matrix can exist (in theory at least) neutral to both forms.

In any cyclical schema there are two paths, and consequently two axial lines with their neutral intermediates; also, in consequence of the manifoldity of cyclic commensurality permitted by the E.T. system, a " distant " progression in one system may be " near " in another.

Different standards of audentity render it difficult to apply a definite criterion, but it would appear that, since in the direct Pythagorean N to $N_{\pm 1}$ the trigonal and tetragonal translations are axial, and also that the quintigrade is equivalent to N to $\pi N_{\pm 1}$, the bigrade or oscillant step of locus is the most extreme of the definitely chiral types.

If this particular translation be also commute, it represents the most extreme tonal translation, although, of course, frequently utilised, and not necessarily so complex in notation as other steps.

The most extreme type is the Commute diametrical, but this is outside the scope of simple translation of Core locus.

The actual modes by which translation is effected do not concern the present examination. The elements affected are either directly or achromatically axial, or simple and compound fluents.

The effect of Pythagorean unigrade translation is the conversion of one of the polar triads into a new core, the degradation of the former core into an element of the new envelope, and the introduction of a new polar triad in the place of the one discarded.

The tonal axes are six in number.

This process may be continued around the cycle, and it is seen that even the extreme sexigrade type possesses two tonal axes, which are transposed in nominance by the translation.

The unigrade Pythagorean is equivalent to an ondecagrade of opposed chirality, but this is of mere academic interest, since cases in which a " key " jumps eleven " flats " or " sharps " must be rare indeed.

Commutative Translation may be thrown into the seminomial-cyclical form by considering a given translation as a part of a particular stage, which may be completed by another step of converse species.

The two companion commute matrices form such stages by whose alternation a cycle can be established. Each alternate stage is of the same species as the nominant, but a quarter cycle in phase as regards locus, in either chiral direction.

Such a cycle is complete in eight " semigrades."

The effect of each definite stage, in such a process, is a bivalence of order, and the discretion of chirality is indicated by the axial and fluent elements of the first, or " pattern " stage.

The process may be abrupt, or can be effected by some such method as adopted in the old " dissolving view " system, where the conservance is temporarily obscured by extraneous conditions while the actual translation takes place.

As regards the criterion between complete translation and some intermediate stage, it may be pointed out that motion in the physical sense can only be in one direction at one time, *i.e.* every particle traces a line, and cannot, while remaining a particle, trace out a space, or volume.

(An infinite number of Euclidian lines side by side might trace a space; this is mere sophistry, but shows how periodicity can appear quasi-dimensional.)

Applying this test, the criterion is the initiation and termination of the whole process at a core triad, which cannot be extended into a complex chord without material alteration of effect. Such a view permits freedom of method in the intermediate stages which may be highly complex, and also of the expansive or contractile types of plural motion which take place in the envelopes of first or second order.

This view facilitates the analysis of complex progressions into terminal and intermediate stages, and, with practice, mere inspection of the terminal stages (whether specifically indicated in notation or otherwise) enables a highly developed work to be grasped as a determinate tonal procedure.

Without the necessary training by experience, it is almost impossible to begin to form an idea of the tonal structure from inspection, and really difficult sometimes to effect by audition, especially where the rhythmic scheme has also to be unravelled.

The Seminomial cycles may be illustrated in Solfa symbols. To save space, only the initial and final stages are quoted.

Fundamental Nominant Dextral

N_0	$N_{\frac{1}{2}}$	N_1	$N_{1\frac{1}{2}}$	. . .	$N_{11\frac{1}{2}}$	N_{12}	$N_{12\frac{1}{2}}$	N_{13}
Soh	Soh	Soh	Fe	. . .	Lah	Lah	Lah	Soh
Me	Me	Ray	Ray	. . .	Fah	Fah	Me	Me
Doh	Te	Te	Te	. . .	Ray	Doh	Doh	Doh

Coincidental Nominant Dextral

Me	Fah	Fah	Fah	. . .	Ray	Me	Me	Me
Doh	Doh	Ray	Ray	. . .	Te	Te	Doh	Doh
Lah	Lah	Lah	Ta	. . .	Soh	Soh	Soh	Lah

Tetragonal Cycle

Fundamental

N_0	$N_{\frac{1}{2}}$	N_1	$N_{1\frac{1}{2}}$	N_2	$N_{2\frac{1}{2}}$	N_3	$N_{3\frac{1}{2}}$	N_4
Soh	Soh	Soh	Fe	Fe	Fe	Me	Me	Me
Me	Ma	Ma	(Re)	De	De	De	Doh	Doh
Doh	Doh	Ta	(Le)	Le	Lah	Lah	Lah	Soh

Coincidental

Me	Me	Fe	(Sa)	Sa	Soh	Soh	Soh	Lah
Doh	De	De	(Ra)	Ma	Ma	Ma	Me	Me
Lah	Lah	Lah	Ta	Ta	Ta	Doh	Doh	Doh

It is interesting to note how equivalents to the permutations of direct and concomitant determinators are introduced by laeval and dextral Pythagorean translation respectively.

Translation from P

To L_I permutes TD
L_{II} ,, PD
L_{III} ,, LD
To T_I permutes the Recessive TD
T_{II} ,, ,, PD
T_{III} ,, ,, LD

The permutes of LD, in both predominant and recessive aspects, are equivalent in E.T. to πJ.

The effect of Pythagorean translation upon the First-order chromes, of the original Hemicyclic Matrix, may be noted as indicating translation in most definite terms, in consequence of the Audentity of those chromes.

Dextral steps affect the predominant Trinomial.

The First Pythagorean Stage diminishes Laeval Blue to W/2.
The Second ,, ,, restores same to π(P : T),
and diminishes centron B to W/2,
and so on.

Laeval steps affect the determinators, *i.e.* the Recessive Tetranomial, until the quadrigrade of translation is attained, when the Predominant Quadrant is affected.

The co-operant species of polarity is here seen, also the fact that the conversion of a first-order chrome into its antinominant cyclic neighbour can be divided into two stages, each of two possible intermediates, *e.g.*—

$$\mathrm{P} : \mathrm{T} \text{ to } \pi(\mathrm{T} : \mathrm{TT})$$

by way of either P : πP, or T : πT.

The fact that the audentity of the Core first-order chrome, and its concomitant, are not attacked until the second stage of Pythagorean translation shows that the dextral and laeval stages may be looked upon as quasi-matrical.

Reasons have been adduced for the exclusion of the Nonomial or Quintigrade elements from the conserved Matrix; but it is to be noted that in some aspects the quasi-matrical character is maintained: viz. in such a passage of successive dyads as:—

$$\left\{\begin{array}{llllllll} \text{Doh} & \text{Ray} & \text{Me} & \text{Fah} & \text{Soh} & \text{Lah} & \text{Te} & \text{Doh} \\ \text{Doh} & \text{Te} & \text{Ta} & \text{Lah} & \text{Soh} & \text{Fe} & \text{Fah} & \text{Me} \end{array}\right\}$$

forming the so-called nonomial scale, which comes under consideration in the Successive aspect.

The independent aspects (which are only theoretical abstractions effected for convenience) of Commutative and Cyclical Translation enable the "Translatant" in either case to be the total range of the other.

Thus, in Commutation, the whole twelve tones of the Dode-

canal are considered as reversed in species, and not merely the conserved Septomial.

Again, in Cyclical (Pythagorean and Antinominant, and their sub-derivatives) Translation, the Translatant embraces the dual validity of the tones, irrespective of Predominant or Recessive discrimination.

Thus, the Translatant in each case is really a " Range " group.

Actually, both forms of Translation are determinative, and thus the Translatant is reduced to its immediate value of the Septomial.

A word may be said as to Justly Intoned Translation, which is not limited to E.T. values. In this case, as also with artificial systems, the liminal temperability of the components may be employed to carry out some extraordinary translations, but it should always be borne in mind that, if determinance is to remain the criterion of a musical materia, such effects will be accepted as quasi E.T. values. This partakes somewhat of the nature of tonal hypnotism and the apparent result is merely to shift, or keep in motion, the centroid of absolute pitch about a small region.

The fact that more or less musical performances can be given, in which insidious sharpening or flattening occurs, without, to the average hearer, detracting much from the musical (not æsthetic) value, shows that if the method of just or arbitrary intonation is to be relied upon as a possible avenue of tonal extension, the liminal capacity, as well as the notational and performatorial technique, will require tightening up to a far greater extent than now generally prevails.

One aspect of Translation, of no great practical importance, remains to be discussed, viz., that between the Polar " Heptads " of the Seriopolar type of Matrix.

The axis of this translation is Riemann's Equation—

$$L_9=T, \quad T_7=L$$

and the consequent extension—

$$L◗=TT$$

The actual translation of polar elements occurs about the mutual quasi and concomitant triads of the Tensor and the Link, the operation, in any case, being a subsidiary element of progression.

SUBSECTION 3

DIRECTIVE AGENCIES

The course upon which chordal evolution has proceeded can be ascertained by reference to the copious, if scattered, evidence of history.

In the main, the chord arises from polyphony, which in itself is a device of human activity.

The various combinations of simultaneous sounds afforded by polyphonic configurations are divisible into those which are rhythmically and organically simultaneous (Chord tones) and those which are not (Autophors).

But the distinction between the objective forms tends to vanish, so that any colligation of simultaneous tones can be regarded in some aspect as a chord.

Out of a fluid " solution " of innumerable forms, due to trial and chance, that have appeared, a slow but certain process of crystallisation has separated out those forming the system of Tonal Chordance.

The selective agency is the determinance of tonality, and the fixative principle is the persistence of hyperacoustic conditions.

Our forefathers knew little of acoustic principles, and generally misapplied them when they abandoned a musical ear for a pseudo-musical intelligence.

We know a little more, perhaps; but our principal gain in knowledge is that of the limitations and defects of our own science.

The process of crystallisation to-day is substantially the same as it always has been; providing the musician of the present age with a veritable heritage of jewels in the form of the most beautiful harmonies, as well as some which cannot be so described.

The limit of the process appears to be the vanishing of definition between the Chord and any chance colligation of tones.

The nomination of chord forms presents a series of relationships diminishing on either side of a predominant aspect.

Thus certain configurations become definitely associated with certain progressions, and thereby acquire a directive tendency.

While recognising the association, it is necessary to avoid any idea of " inevitance." This mistaken idea tended to arise when

reading the older textbooks (although many of these old treatises are found to be larger minded than they appear at first sight).

From these considerations it will be understood that the use of a discord as a direct presentation is quite as much an immediate operation as the use of a compound tint by a painter; the criterion being rationality as to tonal determinance.

Although the directive principle cannot be said to inhere in the chords themselves (since these are selected from practice), it is, however, implicit in the system or aspect of any tonal construct.

Thus the Tertriadal centres on the Prime Triad, the Hemicyclic is general about the Yoke, and the Seriopolar aspect has a bias towards the fundamental laxator.

The antinominant system is far less centric, and aptly presents a fluid relationship for modern requirements.

The aspect we are now about to examine is one that presents many puzzling problems to the student of musical empirics.

In the eyes of the nominalist, it is difficult to see manifoldity in a definite thing that can be named; it apparently conflicts with all the laws of reason that a nominal can split into a group of concepts.

The reason that such an anomaly is possible is to be sought for in the criterion of nominance, and Tonality is not the only science in which the case occurs.

Isomerism and Tautomerism in Chemistry, Dieresis in Rhythm, Umbræ in Mathematics, to mention no more, were all puzzling to nominalists before their real nature was explained.

The anomalous aspect is that presented by the discords of a given matrix.

A tetrad appears:—

(1) Associated with a serial or linked-wing grouping.
(2) As a specific arrangement of the four floating incipient tones enveloping a triad.

Here we have the contrast between a definity and a possibility, *i.e.* a pluri-directed and a centric derivation.

The floating tetrad is somewhat hard to visualise.

If a musician tries to conceive such an aggregation the idea conflicts with all the associations of notation and performance. It is best thought of as a negation of a Core Triad, without specification of components.

TABLE OF TETRADS QUOTED DIATONICALLY

Tensor Series. Groups of:	Tetrad.	F Radical Axis.	Core (Seminomial to the Radicals).	C Radical Axis.
	Te Soh Me Doh	Lah Fah Doh	Lah Fah Ray	Te Fah Ray
(3)	Soh Lah Fah Ray	Te Soh Ray	Te Ta Soh Soh Me Ma	Doh Soh Me
	Ray Te Soh Me	Doh Lah Me	Doh Lah Fah	Ray Lah Fah
(4)	Me Doh Lah Fah	Ray Te Fah	Ray Te Soh	Me Te Soh
(1)	Fah Ray Te Soh	Me Doh Soh	Me Doh Lah	Fah Doh Lah
	Soh Me Doh Lah	Fah Ray Lah	Fah Ray Te	Soh Ray Te
(2)	Lah Fah Ray Te	Soh Me Te	Soh Me Doh	Lah Me Doh

A tetrad is primarily related to a triad in three ways:—viz. as an F or C Radical, or as an Envelope.

In the former case, the lower and upper notes are the respective axes, and in the latter the relationship is non-axial, approximating to parasyntonic.

We illustrate (p. 287) the directive and distributive (as we may call them) aspects by reference to the Scalar radical tetrads, and the same tetrads as they appear in the polar series.

The "Centric" resolution or direction of the (Chord of the Eleventh), *i.e.* Hexad Envelope, is shown by the closed squares.

The directive criterion is the factor outside the chord concept, yet associated and synergetically potent in it (in the sense of calling up a reaction).

It is thus not a matter of Tonality, but (as might have been inferred from the generality of practice) a question of Significance.

In the reverse aspect each Triad could be said to have three envelopes, one for each inversion. Actually, these are one tetradial and two triadal plus axis.

The whole question as to the directive property of chords is enlightened by the genetic principle.

It is evident that when a given configuration arises from a particular phonal colligation, and that alone, its implication is naturally that of its source.

For example, the downward resolution of the "seventh" arose to a great extent from the practice of suspensions upon the sixth. This directive implication was so preponderant that it tended towards a stereotyped resolution.

When, however, tone-artists, such as Caccini and Monteverde, began to use the grouping for the sake of its intrinsic character (as a painter would employ some particular shade or tint) the directivity began to fade away, and would in time entirely vanish were it not for the synergetic bond of association.

It must therefore be concluded that the "older" a chord becomes (and the more it is employed for its purely chromal characteristics) so much does the associated directivity fall off.

The old definition of a discord as something requiring resolution is now tacitly ignored by modern practice employing all configurations in a chromal manner, but the rationale of the old definition remains always true.

The acquisition of freedom in treatment with age does not extend to the insistent acoustical conditions.

Vertication still persists, and even the most modern of ears has not yet attained the art of hearing a Coincidental Series as a whole.

The suggestion of its ancestry persists in every chord, and in an environment savouring of the old atmosphere the directive tendencies reassert themselves.

The natural directive agency of Vertication appears as the bias of chirality in all the "orders." The tendency is to emphasise the F Species, Dextral Polarity, and Educt Determination, and the effect when the Contradeterminator is added to a triad is that of an envelope about a laeval chord. Consequently the direction of progression is towards the opposite "hand" in the orders.

Possibly this tendency has been overestimated from the usual orthodox text-book treatment and the predominance of trichordal harmonic method, but as it arises from the pre-established relationship of the Harmonic Column there is a distinct inclination towards downward resolution of discords, *i.e.* away from the congested upper intervals towards the clear chromes of the triad. Thus the tendency of resolution is catabolic and terminates a series of anabolic processes.

It is to be noted also that any tone in a Series gap is higher in pitch than the member of the corresponding Series.

This is, of course, neglecting achromatic rearrangement.

The essence of scalar cyclicality is achromatism. The more pronounced the recurrency of the octave is, so much the more will the scale divide into two halves, viz.:—

(1) Projectory, from P to (L, πP, T).
(2) Retractory, from (L, πP, T) to WP.

The cacophonic effect of "consecutive fifths and fourths" may be principally attributed to the antinomy of simultaneous projection and retraction. A value cannot be both positive and negative.

In chordance, the directed progression hinges upon the apparently simple motion of Dyads, expansion and contraction.

The simplest case is afforded by the immediate (antinominantal) motion of both or either tones in the central "violet" chrome of the symmetrical envelope (TD : L), which tones terminate the Pythagorean Hemicycle.

T

Such progressions are neutral unless associated with a Series of either species, the natural bias being towards the Fundamental.

The tertriadal view impresses the importance of the semi-octave, and its nearest resolution by convergence and divergence.

The standard diphonic convergence

$$\frac{\mathrm{L}}{\mathrm{TD}} \text{ to } \frac{\mathrm{D}}{\mathrm{P}} \text{ (diminished B to G)}$$

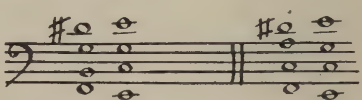
and the corresponding divergence

$$\frac{\mathrm{TD}}{\mathrm{L}} \text{ to } \frac{\mathrm{P}}{\mathrm{D}} \text{ (augmented R to Y)}$$

is a most obvious factor of simple harmonic treatment.

The general invertibility of chordal resolution meets with a usual exception in the case of the Bi-contradeterminator.

The first example shows this chord, which does not sound nearly so well when inverted as the similar melodic progression shown in the second example.



These chordal conditions, by emphasis of the chord and autophor distinction in melodic form, have a distinct effect upon the directive tendency in phonal succession.

The effect of Vertication is manifest in all tonal procedure, but the most striking evidence of its power is the fact that both Modes (Major and Minor) are used together and singly in the same manner.

The name " Leading Note " for the Tensor Determinator also speaks for itself. One does not hear the flat submediant called by any characteristic name, although in most cases it falls to the dominant.

The Chrome R comprised between T and P contains the two Ultra-core Envelope spaces, *i.e.* the antithetical /Æ and E/. It is therefore of ambi-species.

The direction of resolution as evidenced by the chains of sevenths met with in works from the Haydn to Schumann period, the limited nonomial scale, *i.e.* that containing Ta or Fe, used in contrary motion with a diatonic scale, and the contrast between " fluent " treatment of second-order chromes in contrast with

the axial methods in the case of first order, all point to the directive effect of vertication in the several orders.

The vanishing of the "leading note" effect and other alterations seen on the adoption of the Hemicyclic (Modal) treatment, indicate the genetic course of the Tertriadal aspect.

In the Modal system the important chrome is that of First order; the Second-order chromes follow the species that happens to fit the scale.

The dextral hemicycle provides six principal and one imperfect mode, in the latter the neutral hemichrome W/2 stands for a scalar B or R. The number of Modes is thus seven, presenting two "species" of reading; total 14.

The composite aspect of these two species (refer to the Minor Mode aspect of the Coincidental Species) brings us to the Ortho- and Hypo-, or Authentic and Plagal, varieties of Modes, well known in practice.

The important point is to note how the Axial tendencies of First-order elements conduce towards a fluidity of practice in Second Orders, the bias of the Series then confers a chirality upon the cycle.

It is not without interest to consider the change of concepts that results from the arguments laid down in the preceding remarks and chapters generally.

The empiric concept of a "chord" yields place to a concept of the Series, in two species, the Fundamental very obvious, and the Coincidental apparently artificial, but founded upon real conditions.

The idea of progression from Discord to Concord is replaced by the notion of Envelope to Core; the concept of the Envelope as a series of spaces being somewhat strange at first.

The conditions of tonality are presented as a set of possibilities, whose interdependence tends towards a mutual moulding. The three principal aspects of arrangement have been discussed under the names of Tertriadal, Hemicyclic, and Seriopolar, but there is nothing to limit the aspects to three.

The requirements of what Helmholtz calls the "æsthetic" conditions lead to the selection of the contributive conditions, with total results often differing very widely; but the conditions themselves persist because they are founded upon the principles of Hyperacoustics, viz. the actual essentials of musical manifesta-

tion. There is nothing in this view that precludes development, nor, on the other hand, can any predictions be made.

In making observations of the phenomena of music for the purpose of obtaining scientific data to work upon, many extraneous conditions have to be " weighed out " or allowed for.

It is well to bear in mind, that the wealth of modern methods available at the present time for composition and performance, especially the vast orchestral organism and the mechanical players, not only affords many facilities for the development of pure musical manifestation, but also for a kind of aural hypnotism and musical sleight-of-hand.

It is no business of the scientific observer to sit in judgment upon works of art, but some of the extraordinary examples of so-called advanced music invite the question as to where the line between artist and prestidigitateur is to be drawn.

SUBSECTION 4

TYPES OF AGGREGATION AND CONCLUDING REMARKS

It frequently happens that the investigation of a phenomenon, both on inquiry into its historic evolution, and also as a theoretical study, yields little or no satisfactory result.

Every view-point presents groups of antithesis, and no single derivational or determinantal theory appears capable of solving the problems presented.

At such a time a multi-radical analysis is often of value.

By seeking out the main derivational conditions and developing therefrom, regardless of the apparent complexity, it is often found possible to trace the evolutional course and theory of action.

In studying the problems of acoustics the method has been found to be of value. The reason for extending it to the present investigation is the physiological fact that musical activity concerns both efferent and afferent organs of the body.

Musical procedure is thus conditioned by two entirely different sets of organic operations, viz. the " Producer " set (which includes all generative and controlling agencies) and the " Auditory," *i.e.* the ear.

In the case of the human organism, the first class has become

extended from the original vocal organs to means of instrumental control (fingers, lips, lungs, and indirectly the whole muscular system). The orchestral player and the organist employ many actions, the trick player shows the extent to which they can be carried if required.

But in the case of audition, the ear remains the sole seat of hearing. Up to the present, little has been done to hear by any other avenue than the auditory nerve: a very slight derangement of the auditory organs serves to cut the sensory system off from the external world of sound.

It will be noted that we are considering the physiological aspect, and the psychological sequence of perception and conception does not enter. But the mental processes are conditioned by the circumstances under which actual manifestation occurs, viz. physiological acoustics, so that the composer of music may be regarded as employing both processes, conceptually, at least. The process consists in the formulation of concepts and the selection therefrom, two separately derived operations which may operate simultaneously.

In consequence of this biological duality the human mind is concerned with two aspects. The two sets of perceptive elements may fuse, interchange, and interact in such a way as to produce an appearance of unity, which upon attempted analysis results in incompatibilities, both practical and theoretical, only soluble by reference to the duality of the physiological conditions under which we musically exist.

These two divisions are somewhat difficult to describe, but the tendencies of each may be traced as follows:—

Formulative.—The concept is that of extension, *i.e.* a unidimensional idea of pitch apart from any tonal concept.

By the use of the illustration of spacial extension, we can see how figures are delineated, imitated, linked, segregated, aggregated, etc., according to all the devices of rhythm.

It is possibly difficult to imagine pitch without tonality: the generality of the E.T. twelve-toned scale is a case.

The idea is the Shape of the images considered apart from the actual scale employed, somewhat in the way in which the shape of a tune can be recognised on an out-of-tune instrument, apart from any question of musical quality.

Shapes may look most alarming in tonality, but if the ear

recognises them, their existence is justified. This may be the justification for much that is considered extreme in modern harmony practice.

This aspect is derived from the "producer" set of organs, and it is in this direction that great skill and dexterity in production usually tends. It has been noticed that a loss of auditory power usually results in a departure from melodic effect, while technique and formulative skill remain.

The second element or aspect (the Auditive) is what may be called the Chromal or Nominative tendency.

Its source is the ear, consequently the nominative form tends towards the harmonic structure derived from the Series, and its envelope.

The ear does not grasp pitch as a dimension: that is evident upon introspection and observation of the young: but it appreciates it in some such way as that in which colouring stands to drawing in delineative and pictorial art.

Colour is not a spacial or protensional sensation. Its intension is independent of its character (we are not now referring to the aberration and saturation conditions of the eye), but it is a specific quality whose description has to be associative with external conditions.

If we try to describe the spectrum "regions" instead of definite colours, we should find it to consist of two principal divisions, the Warm and Cold, separated by the maximum intensity of yellowish green. The regions would read:—

Hypermelan, Pyrochrome, Chrysochrome, MEGAPHOS, Chlorochrome, Cyanochrome, Hypomelan.

The range of auditory pitch could be described in a somewhat parallel manner, the regions being:—

Throb, Bass, Canto-tenor, Mezzo-alto, MIDDLE, Treble, Acerb, Acute, Chirp.

These do not suggest form or dimension, but are more of the type of mass or "touch" sense.

The spacial element is borrowed, to a great extent, from the Producer Aspect.

The production of instrumental sound has, in general, very distinct spacial associations.

Reinforcement of Sound - Ma (92)

185 Low intensity level - main floor underneath balcony, rear of balcony.

Must aim to establish uniform sound intensity over the entire seating area.

186 loudspeaker - microphone

From the harp, organ-pipe, and bass strings we have the vertical dimension (the conventional ideas of up and down are reversed, as in early Greek times). The keyboard presents a right- and left-handed notion, and violins, flutes, trombones, etc., introduce the "towards and from" idea.

The voice and allied lip-work in blowing harmonics on tubes are less spacial, and more intensive, in character.

The earliest acoustical researches were concerned with what might be called the "dimensional" aspect of frequency, *i.e.* the lengths of strings and tubes.

At a later date the history of acoustics shapes a course partly directed by tonal conditions which are not so simple.

The natural result of the tonal aspect has been the nomination of each aspect, while the spacial view is concerned with the formation of figures in time.

Symbolic notation, such as the "Tonic-Solfa," is most helpful where a nominate basis is required, and is, therefore, of particular value in singing. Semi-graphical notation, such as that on the Staff, presents groups to the eye which aid the fingers on the keyboard, strings, or tubes to spacially arrange themselves.

Every tone has two aspects, viz. its nominance and its teleological significance, *i.e.* what it is, and where it is going. Both expressions are composed of a multitude of contributions from the hyperacoustic conditions, and range about a limited number of predominant forms.

The great difference in the nature of the muscular operations involved in playing various instruments is the reason that no system of "spacial" generality has been evolved.

It certainly might be urged against the association hypothesis that a performer on more than one instrument rarely becomes confused, but this objection only emphasises the abstract nature of the association.

There is no doubt that the exercise of power and agility in the "gymnastic" aspect of performance is very enjoyable to the performer and those in sympathy. The results attained often seem wonderful in respect to the limitations of the hands, etc.

The setting out of the spacial concept without the aid of Tonality is difficult, if not impossible.

The primary idea of Core and Envelope can be applied to

any set of relationships. If any "notes" approach so that their "parasyntonic regions" come into contact, the gaps necessarily vanish, and a quasi-flexionic continuity results.

A discrete scale necessarily follows the adoption of a Core and Envelope system, and the chordal arpeggio confers a tonal character upon it.

The "dodecanal scale" in E.T. is certainly unlocated, corresponding somewhat to Dr. Eaglefield Hull's idea of a Dodecuple Scale, but in practice it will be noted that the Chromatic Scale is either used tonally, or for the sake of its own "passing note" effect.

An amorphous scale (di-tetrad heptonic) embraces the following range of tones:—

Le	Tee	De		Ma	Fa	Sa
Lah :	Te :	Doh :	RAY :	Me :	Fah :	Soh
La	Ta	Da		Mee	Fe	Se

Such a scale might be given by a flute with holes equally pitched between:—

LA			RAY			SE
Open	1	2	3	1	2	3

Into such a system, the octave and its half enter. We might disguise the fact by changing names, such as "double-wave" for octave, but the tonality is there.

The chordal system of tonality gives rise to the so-called "Third-building," *i.e.* the Heptad 1 : 5 : 3 : 7 : 9 : 11 : 13, in which each gap might be named.

From this practice, it has been inferred that "Fourth-building" and other similar constructs are rational. Possible they certainly are, with their own special effects, but not to be justified tonally unless as a series of Applements R.

The reason for their employment is rather to be sought for in a transposition of low-pitched wide intervals, which, from acoustic reasons, are preferable in effect to the "Thirds."

Figured Bass methods would be applicable to non-tonal constructs.

From the foregoing considerations, it is obvious that the concept of a "Hapta-metric" (Touch measurement) aspect of scale division involves very complex problems.

The basic idea of tonal determinance is the recognition of intervals as entities, as opposed to the objective nature of tones themselves.

An interval can be recollected, while a particular tone soon fades in the average memory.

An interval can be transposed over a wide range of pitch without much variation in character, and this implies that a discrete scale of contingent intervals can easily be stepped out.

On this spacial system, the conditions of chordance determine the actual form of simultaneous and successive practice. The effect of chordance is almost like colouring a line drawing, in that it is flexible to the will of the artist.

The general form of a Phonon is either Scalar or Arpeggial, and the coherence of certain predominant chord-forms permits of "chordal exchange," viz. arriving at one component and quitting another of the same chord. This is perplexing to the eye, but the ear follows it readily.

The two chord forms upon which all construction is built are the Core Triads, and their Quasi-homochromal Envelopes.

Upon this system of structure, by means of Autophors, and non-simultaneity of time, the formulæ are built up.

How they are employed and organised into complete works of music is beyond the scope of the present inquiry.

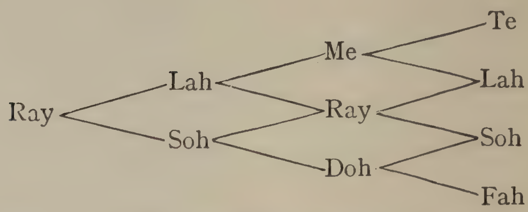
It is interesting to compare the scalar step from Core note to Envelope note, with the Measure in Rhythm, and the larger Serial intervals may be compared with the Unicepts and Gyroids (Phrases and Sentences) leading up to the organic system.

The scalar systems built upon the Tertriadal, Hemicyclic, and Serial aspects of tonal relationship can be recognised in all kinds of musical works. Indeed it is very interesting to watch the dexterity with which a composer employs them and the fine shading from one aspect into another.

We may recapitulate the derivation of the Tertriadal from the Prime Triad and its companions in the Trinomial, which aggregation is limited by the pseudo-equation of first and second chromal relationship.

The Hemicyclic, it will be recalled, starts from the tone Ray, and tunes outwards in Pythagoreans. We thus get the grades

of audentity corresponding with Modal Scalar practice as follows:—



The "Serial" Scales are most obvious on instruments that yield "upper partials" (narrow tubes).

Taking the Tensor as the natural fundamental of the series (since it yields tones approximating best to the "Natural Scale"), we have, within octave range, the following relations, which are capable of achromatic reduction to scale form (the gaps should be noted):—

1. T T
2. T J T
3. J T TD J
4. T TD J (L) T
5. TD J (L) T LD TD
6. J (L) T LD TD (P) J
7. (L) T LD TD (P) J (PD)

The last of these gives the so-called Scriabine Scale approximately:—

Fah : Soh : Lah : Te : De : Ray : Me

It is not at all disagreeable, and is used by Scriabine with considerable effect, although fragments may be noted in much earlier works.

But unless the instrumentalists use justly intoned instruments and the special Serial Notation not yet invented, its claims to seriality are somewhat far-fetched; the effect resembling a kind of modified "Dorian" mode.

Thus the investigator is led to the gateway of Successive Tonality. The problems of the Simultaneous aspect cannot be restricted to the sphere of discussion to which the present division of inquiry is limited. In particular, the observer is brought into

touch with the basic Formulation and Figuration which pervades all classes of musical manifestation.

This brief glance forward shows how complex the actual conditions behind musical presentation really are.

No cut-and-dried scientific method can successfully deal with a living and growing manifestation of conscious existence, but at the same time, there is yet a wide scope for a critical survey of Material and Method, such as has been attempted in this division.

In conclusion, it is advisable to reiterate the primary statement that the experience of Tonality does not admit of abstraction.

Abstraction is performed for analytical purposes, in order to apply acoustical criteria to an admittedly hyperacoustical set of problems, with what success remains to be seen.

It cannot be expected that such an abstract consideration as "Simultaneous Tonality" will be evident, or even entirely comprehensible, until the Successive view is dealt with, and this in its turn demands some knowledge of Rhythm and Organisation, etc.

In fact, Hyperacoustics is a subject that must be necessarily viewed from the total standpoint of musical manifestation.

The empirical student of harmony and counterpoint is working in practical musical fields, and thus gains more than a mere empty knowledge of names and guiding principles.

It is possible, and even probable, that this abstract analysis will in time be found dispensable.

But it will be admitted that it is a convenient stage in the process of assimilating the facts and principles of Hyperacoustics (to claim no more) if always the premises are found to be correct and the logical processes rational.

More than this cannot be claimed, and the work will stand or fall by its actual merits on this basis.

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